DETERMINATION OF THE ABSOLUTE LENGTH OF EIGHT ROWLAND GRATINGS AT 62° FAHR.

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In January of the present year, the writer received from Professor Rowland eight diffraction gratings, with the request that the absolute distance between the terminal lines should be determined with great precision, with reference to their use in the measurement of absolute wave lengths. Four of these gratings were five centimeters in length, and the remaining four had a length of one decimeter.

The first step in the standardization of these gratings consisted in the establishment of definitely determined units of comparison upon bars of speculum metal having the same composition, and presumably the same coefficient of expansion as the metal upon which the gratings were ruled.

In any comparison of units of length it is important that the conditions under which the observations are made, shall differ as widely as possible. This diversity is not conducive to a close agreement of the individual results obtained, but the final mean will be nearer the truth than when the comparisons are repeated under substantially the same conditions.

Accordingly, in this series of observations, aliquot parts of a meter at 62° Fahr. were laid off upon three bars of speculum metal. They were made by Mr. Brashear of Pittsburgh.

Bar No. 1, which is 52 centimeters in length, has three sets of metric and three sets of English graduations, designated $\overset{a}{S}_0$, $\overset{b}{S}_0$, $\overset{c}{S}_0$. The metric graduations consist of 50 centimeters with triple lines for the decimeters. The lines of $\overset{a}{S}_0$ have about twice the width of the lines of the gratings; those of $\overset{b}{S}_0$ are a little finer than the average width of the lines of the gratings; while those of $\overset{c}{S}_0$ have about the same width.

Bar No. 2, designated $\dot{S}_1$ is 23 centimeters in length, and is graduated to 20 centimeters. The width of the lines is somewhat less than that of the finest lines of S_0.

Bar No. 3 is 27 centimeters long, and has 20 centimeter graduations. The lines have about the same width as those upon $\dot{S}_1$ This standard is designated $\dot{S}_2$.

The ultimate standard to which all comparisons are referred in this discussion is the bronze yard and meter $\dot{R}^2_2$ described in my paper, Studies in Metrology; Proceedings of the American Acadedy, Vol. XVIII., pp. 287–407. The steel standard $\dot{R}^2_1$, also described in this paper, was originally an end-measure only. It was purchased in Paris of the celebrated mechanician M. Froment. The line graduations were added by the writer. This standard is now the property of the Sheffield Scientific School of Yale College.

The relations of standards $\dot{R}^2_2$ and $\dot{R}^2_1$ to the Metre des Archives and to the Imperial Yard of Great Britain, described in my memoir, are derived from all the data at hand when my memoir was published. Since that time additional data bearing upon the subject have been obtained. It will be remembered that the relation was derived from a yard and meter laid off upon a bar of bronze having the same form and dimensions in cross section as the Imperial Yard.

A *B* *C*

Representing the yard by the line *AB*, and the meter by the line *AC*, the operation consisted:

(*a*) In the determination of the length of *AC* in terms of the Metre des Archives, A_0 through a comparison with a meter upon copper traced and standardized by Tresca and with a meter upon brass belonging to the Stevens Institute and compared with Type I of the International Bureau of Weights and Measures at Breteuil by Dr. Benoit.

(*b*) In the determination of the length of *AB* in terms of the Imperial Yard through comparisons with "Bronze 11" belonging to the United States Coast Survey and with a yard upon brass belonging to the Stevens Institute which has been compared directly with the Imperial Yard by Mr. Chaney, Warden of the Imperial Standards.

(*c*) The determination *BC* in terms of either *AB* or *AC*.

Under these divisions the following additional data have been obtained:

(*a*) In my paper, Studies in Metrology, Proceedings of the American Academy, Vol. XVII., p. 382, the following relations are given:

From a comparison of the bronze meter R_2 and the steel meter R_1 with the Tresca meter T, designating the Metre des Archives by A_0, we have:

$$\overset{a}{R}{}^{2}_{1}-A_0=-3.2\mu.$$

$$\overset{a}{R}{}^{2}_{2}-A_0=+1.5\mu.$$

From a comparison of the meters R_1 and R_2 with the Stevens Institute meter designated C S,

$$\overset{a}{R}{}^{2}_{1}-A_0=-2.5\mu.$$

$$\overset{a}{R}{}^{2}_{2}-A_0=+1.1\mu.$$

The values of the coefficient of expansion for the bars T, C S, R_1 and R_2 were found as given on page 380, viz.:

For T,	coefficient	for	1°C.	$=16.18\mu$.
For C S,	"	"	"	$=17.60\mu$.
For R_1,	"	"	"	$=10.11\mu$.
For R_2,	"	"	"	$=17.17\mu$.

The length of the meter C S was derived from the provisional relation communicated by Dr. Pernet, viz.:

$$\text{C S at } 0°\text{C.}+310.0\mu=A_0.$$

The coefficient of expansion of this metal determined at Breteuil had not been communicated at the time of the publication of my paper. It was, therefore, necessary to use the value given above, viz., 17.60μ in the reduction to 16.67° C. or to 62.0° Fahr.

In February last, I received from Dr. Pernet not only the exact observed relation between C S and Type I of the Bureau, but also the adopted value of the coefficient for this bar. The following are the relations communicated:

$$\text{C S}+310.7\mu=A_0.$$

Coefficient of C S$=17.71\mu$ for each degree C.

Substituting these values for those originally employed, we have the following relation between R_2 and A_0:

From the Tresca meter, $\overset{a}{R}{}^{2}_{2}-A_0=+1.5\mu$.

From meter C S [Rogers], $\overset{a}{R}{}^{2}_{2}-A_0=+1.1\mu$.

From meter C S [Benoit], $\overset{a}{R}{}^{2}_{2}-A_0=+2.3\mu$.

(*b*) In the reduction of the comparisons of the yard $\overset{a}{R}{}^{2}_{2}$ with "Bronze 11," the relation

"Bronze 11"+.000088 inch=Imperial Yard,

as determined by Hilgard and Chaney, was employed. From the Report of the Standard Department for 1883, it appears that Pierce found the relation,

"Bronze 11"+.000022 inch=Imperial Yard.

Substituting the latter relation for the former, I find:

From mean of observations by Mr. Edwin Smith and myself,

$$\overset{a}{R}{}^{2}_{2}-0.2\mu=\text{Imperial Yard}.$$

From the relation between C S and the Imperial Yard given by Mr. Chaney,

$$\overset{a}{R}{}^{2}_{2}-0.9\mu=\text{Imperial Yard}.$$

For the present following relations will be adopted:

For the Meter.	For the Yard.
$\overset{a}{R}{}^{2}_{2}-1.6\mu=A_0$.	$\overset{a}{R}{}^{2}_{2}-0.6\mu=Y$.
$\overset{a}{R}{}^{2}_{1}+2.3\mu=A_0$.	$\overset{a}{R}{}^{2}_{1}-2.2\mu=Y$.

In the determination of the absolute length of $\overset{a,b,c,}{S_0}$, $\overset{a}{S}_1$ and $\overset{a}{S}_2$, it was considered advisable to make the comparisons not only with the subdivisions of the bronze meter, but also with those of a steel meter, for the purpose of obtaining an independent check upon the work. In January, 1884, the writer constructed three meters upon bars of Jessup's steel, of the form described on p. 290 of the paper referred to above. Yard and Meter B was made for Professor Brockett of Princeton College; Yard and Meter A_4 was sent to

Professor Rowland, while Yard and Meter R_3 was retained for my own use.

Much attention has been given to the determination of the relation between the length of these three standards in terms of bronze R_2, especially with reference to the constancy of the equation. The equation between the Brockett Standard B and R_3 from observations in 1883, was found to differ by a sensible amount from the relation found in 1884. Again, the relation between R_2 and R_3, determined in the early part of 1884, differed materially from that found in the latter part of the same year. In order to obtain an additional test with regard to the constancy of a determined relation between these three standards, Professor Rowland kindly returned to me Yard and Meter A_4, in order that I might be able to repeat the comparisons with R_2, made in 1884. It does not seem advisable to burden this paper with the mass of details involved in these comparisons. In order, however, that the data may be furnished for the computation of the probable errors of observation, the separate results of the comparisons of R_2 with A_4 are given. They are as follows:

Results of the various determinations of the relative coefficient of expansion between bars R_1, R_2, R_3 *and* A_4.

Relative Coefficient Between
R_2 and R_3.

Limiting Times of Observations.	Weights.	Coefficient.	
1884. Feb. 5, to Mar. 5,	1	6.90μ	from meter.
" " " "	1	6.91	from yard.
" Nov. 16 to Dec. 21,	$\frac{1}{2}$	6.79	from meter.
" Dec. 21 to Dec. 31,	$\frac{1}{2}$	6.99	from meter.
1885. Jan. 20 to Feb. 15,	2	6.93	from meter.
" Feb. 16 to Mar. 5,	$\frac{1}{2}$	7.01	from meter.
" " " "	$\frac{1}{2}$	6.87	from yard.
		Mean=6.92μ	

Relative Coefficient Between
R_1 and R_3.

Limiting Times of Observations.	Weights.	Coefficient.	
1884. Feb. 5 to Mar. 5,	1	0.28μ	from meter.
" " " "	1	0.19	from yard.
" " " "	1	0.17	from meter.
" " " "	1	0.18	from yard.
		Mean $=0.20\mu$.	

Coeff. of $R_2=17.17\mu$. Coeff. of $R_1=10.11\mu$.
Hence, Coeff. of $R_3=10.25\mu$. Coeff. of $R_3=10.31\mu$.

Relative Coefficient Between
R_2 and A_4.

Limiting Times of Observations.	Weights.	Coefficient.	
1884. Feb. 5 to Mar. 5,	1	6.90μ	from meter.
" " " "	1	6.91	from yard.
1885. Jan. 20 to Feb. 15,	1	6.93	from meter.
" Feb. 16 to Mar. 5,	$\frac{1}{2}$	7.01	from meter.
" " " "	$\frac{1}{2}$	6.88	from yard.
		Mean $=6.93\mu$.	

Relative Coefficient Between
R_1 and A_4.

Limiting Times of Observations.	Weights.	Coefficient.	
1884. Feb. 5 to Mar. 5,	1	0.22μ	from meter.
" " " "	1	$+0.19$	from yard.
" " " "	1	$+0.17$	from meter.
" " " "	1	$+0.18$	from yard.
		Mean $=0.19\mu$.	

Coeff. of $R_2=17.17\mu$. Coeff. of $R_1=10.11\mu$.
Coeff. of $A_4=10.24\mu$. Coeff. of $A_4=10.30\mu$.

By combination we have:

Absolute coefficient of R_3	$=10.25\mu$.	Weight 1	From R_2 and R_3
" " "	$=10.24\mu$	" 1	From R_2 and A_4
" " "	$=10.31\mu$	" $\frac{1}{2}$	From R_1 and R_3
" " "	$=10.30\mu$	" $\frac{1}{2}$	From R_1 and A_4

Whence, for the absolute coefficient of Jessup's steel for each degree centigrade we have:

10.265μ.

For the relation between R_2 and R_3 the value 6.91μ has been adopted in the reductions which follow.

COMPARISON OF METERS.

From observations between Feb'y 5 and March 11, 1884.			From single observations between March 12 and March 21, 1884.		
No. obs.	Y 61*	R_2—A_4	Date.	Y 61.	R_2—A_4.
	°	At 16.67° C.		°	At 16.67° C.
3	+30.55	$+4.1\mu$	Mar. 12	+15.39	$+3.8\mu$
2	+27.48	+1.7	" 14	15.97	+4.9
4	+24.01	+4.4	" 15	14.32	+2.7
4	+14.16	+0.2	" 16	11.74	—0.1
2	+ 5.92	+5.3	" 17	10.87	+1.7
6	+ 2.40	+1.8	" 18	12.31	+3.2
5	+ 1.85	+3.0	" 19	12.62	+4.9
4	—3.24	+1.8	" 20	13.63	+5.5
3	—4.99	+2.5	" 20	11.98	+2.9
3	—7.91	+3.7	" 21	11.88	—0.5

* The readings of the thermometers Yale College, Y 61, and Baudin 8614, have been reduced to the standard adopted at the Thermometric Bureau of the Observatory of Yale College.

FROM OBSERVATIONS

Between January 20 and February 15, 1885.

Date.	Baudin 8614* °	$R_2 - A_4$. At 16.67° C.	Date.	Baudin 8614 °	$R_2 - A_4$. At 16.67° C.
Jan. 20	15.96	—2.0μ	Feb. 7	30.08	+ 1.6μ
" 20	15.17	+2.1	" 8	29.38	+0.8
" 30	13.70	—0.2	" 8	29.80	—1.3
" 30	15.90	—3.8	" 12	28.30	—1.8
" 30	17.52	+0.8	" 13	27.55	—0.2
" 20	16.54	—1.6	" 13	28.64	+2.4
" 28	18.34	+2.0	" 13	28.54	+2.6
" 20	26.54	+0.2	" 13	28.40	—1.2
" 21	27.21	—0.1	" 14	27.74	+4.1
" 28	26.19	+0.7	" 15	27.14	—2.0
" 28	29.43	+2.2	" 15	27.84	+4.2
" 28	28.78	—1.7	" 15	28.80	+4.7
" 28	28.34	+2.4	" 15	29.84	+0.0
" 20	28.93	—1.1	Jan. 21	41.91	+1.1
" 27	31.98	—1.1	" 21	40.13	—1.7
			" 21	41.25	—2.0
			Feb. 1	47.79	+3.1
			" 1	47.77	+1.5
			Jan. 25	78.65	—1.5
			" 26	80.14	+2.7
			" 26	81.42	+2.8
			" 25	81.82	—0.2
			" 26	81.76	+3.0

*The readings of the thermometers Yale College, Y 61, and Baudin 8614, have been reduced to the standard adopted at the Thermometric Bureau of the Observatory of Yale College.

COMPARISON OF YARDS.

From observations between Feb'y 5 and March 11, 1884.

No. obs.	Y 61.	R^2_2—A_4.
	°	At 16.67° C.
4	+30.56	+ 0.7μ
7	+25.80	+ 7.3
7	+24.53	+ 5.4
2	+14.89	+12.8
5	+ 5.88	+ 0.2
3	+ 4.45	+ 3.0
3	— 0.23	+ 6.2
6	— 6.42	+ 1.2
4	— 7.55	— 0.7
5	— 7.89	+ 0.2

From observations between March 12 and March 27, 1884.

Date.	Y 61.	R^2_2—A_4.
	°	At 16.67° C.
Mar. 12	15.39	+3.7μ
" 14	15.97	+0.4
" 15	14.32	—0.8
" 16	11.74	+4.5
" 17	10.89	—0.7
" 18	12.31	—2.4
" 19	12.62	+0.8
" 20	13.63	+0.5
" 20	11.98	+6.7
" 21	11.88	—4.9
" 23	14.48	—0.9
" 24	21.22	+1.1
" 25	10.88	—2.7
" 25	12.83	—2.9
" 26	14.56	+5.0
" 26	14.90	+7.4
" 14	14.96	+1.1
" 27	15.03	+0.4

FROM OBSERVATIONS

Between January 12 and February 24, 1885.

Date.	Baudin 8614.	R^2_2—A_4	Date.	Baudin 8614.	R^2_2—A_4	Date.	Baudin 8614.	R^2_2—A_4
	°			°			°	
Jan. 12	53.69	+6.4μ	Feb. 17	30.30	+6.8μ	Feb. 20	27.84	+5.6μ
" 12	57.81	+2.5	" 17	29.89	+4.7	" 21	27.56	+5.1
" 13	57.04	+2.3	" 17	29.80	—0.5	" 21	26.42	+1.9
" 14	51.22	—2.4	" 17	29.06	+5.7	" 21	27.88	—1.6
" 14	51.48	—3.6	" 18	27.54	+3.5	" 22	27.70	—1.5
" 14	53.87	—0.3	" 18	27.40	+2.2	" 23	27.72	—0.2
Feb. 16	29.54	+5.7	" 19	26.84	+0.6	" 24	27.36	+2.3
" 16	29.64	+5.3	" 20	27.46	+2.7			
" 16	30.16	+6.5						
" 16	30.98	+2.2						

Omitting the details of the comparisons of R_3 with $\dot{R}^2_2$ and R_1 we have for the mean results the following relations between $\dot{R}^2_2$, $\dot{R}^2_1$, R_3 and A_4. The Imperial Yard is designated by Y.

METERS.

Date of Comparisons.	Comparison with	Equations Between A_3 and A_0	Weights.
1884, Feb. 5 to Mar 11.	R_2 (meter)	$R_3+4.0\mu=A_0$	1
" " " "	R_2 (yard)	$R_3+4.1\mu=A_0$	1
" " " "	R_1 (meter)	$R_3+2.4\mu=A_0$	1
" " " "	R_1 (yard)	$R_3+2.7\mu=A_0$	1
" Nov. 16 to Dec. 21.	R_2	$R_3-1.2\mu=A_0$	$\frac{1}{2}$
" Dec. 21 to Dec. 31.	R_2	$R_3+2.2\mu=A_0$	2
1885, Jan. 2 to Feb. 15.	R_2	$R_3+1.9\mu=A_0$	3

Adopt. $\mathbf{R_3+2.4\mu=A_0}$.

YARDS.

1884, Feb. 5 to Mar. 11.	R_2 (yard)	$R_3+1.0\mu=Y$	1
" " " "	R_2 (meter)	$R_3-1.4\mu=Y$	1
" " " "	R_1 (yard)	$R_3-0.6\mu=Y$	1
" " " "	R_1 (meter)	$R_3-0.8\mu=Y$	1
1885, Jan. 12 to Feb. 16.	R_2	$R_3-0.5\mu=Y$	1

Adopt. $\mathbf{A_3-0.5\mu=Y}$.

METERS.

Date of Comparisons.	Comparison with	Equations Between A_4 and A_0.	Weights.
1884, Feb. 5 to Mar. 11.	R_2	$A_4+1.2\mu=A_0$	2
" Mar. 12 to Mar. 21.	R_2	$A_4+1.3\mu=A_0$	2
" Feb. 5 to Mar. 5.	R_1 (meter)	$A_4-0.4\mu=A_0$	1
" " " "	R_1 (yard)	$A_4-0.0\mu=A_0$	1
1885, Jan. 10 to Feb. 15.	R_2	$A_4-0.9\mu=A_0$	3

Adopt. $\mathbf{A_4+0.2\mu=A_0}$.

YARDS.

1884, Feb. 5 to Mar. 11.	R_2	$A_4+1.8\mu=Y$	2
" Mar. 12 to Mar. 27.	R_2	$A_4+1.3\mu=Y$	2
" Feb. 5 to Mar. 5.	R_1 (yard)	$A_4+1.4\mu=Y$	1
" " " "	R_1 (meter)	$A_4+0.2\mu=Y$	1
1885, Jan. 12 to Feb. 24.	R_2	$A_4+1.8\mu=Y$	3

Adopt. $\mathbf{A_4+1.1\mu=Y}$.

It may be said that the disagreement in the results given above, is not much greater than the probable error of observation, except in the case of the equation $R_3 - 1.2\mu = A_0$. Here, the agreement between the separate comparisons forbids the assumption of accidental errors as the cause of the variation. Besides, the disagreement between the different values of the relation $R_2 - B$, omitted here on account of the incompleteness of the first set of observations, is even greater than for $R_2 - R_3$.

In all of the comparisons made since 1884, especial pains have been taken to eliminate the error arising from the failure of the thermometer when placed upon the upper surface of the standards to indicate the real temperature of the metal. Even when a thermometer reading seems to be stationary, it has a *drift* up or down. The direction of this drift is always noted, and the aim has been to combine the observations in such a manner as to balance the errors due to this cause. Very gradually, however, the conviction has been forced upon me that when the temperature of a considerable mass of metal is obtained from the reading of a thermometer placed upon its surface, errors of long period are always introduced which escape detection in a short series of observations, but which manifest themselves in a long series in passing from winter to summer and *vice versa.* This view must at present be taken as a working hypothesis; but if it should prove to be correct, it will have an important bearing upon the work already done in metrology.

In order to test this theory it is absolutely necessary that the comparisons shall be made in a room in which an apparently steady temperature can be maintained for a long period of time, and in which the changes which occur are very slow. Otherwise, it would be impossible to separate the errors of short period from those of long period. Through an appropriation from the Rumford Committee of the American Academy of Arts and Sciences, a comparing room has been built beneath the rotunda of the Observatory which nearly fulfills the required conditions. The full account of the work undertaken by Mr. McRae and by the writer, under this appropriation, will appear in a report to the Academy, but by permission the results of the comparisons between R_2 and R_3 are given here. They are as follows:

Date.	T.	R^{a}_2—R_3	Date.	T.	R^{a}_2—R_3	Date.	T.	R^{a}_2—R_3
	°	At 16.67°c.		°	At 16.67°c.		°	At 16.67°c.
May 25	9.52	+6.6μ	Jan. 1	10.73	+5.1μ	Jan. 15	13.04	+4.0μ
" 25	9.32	+3.2	" 1	10.83	+3.2	" 15	13.37	+2.8
" 25	9.61	+2.2	" 1	10.89	+3.6	" 16	13.30	+3.7
" 25	9.77	+5.3	" 1	10.80	+5.7	" 16	13.30	+3.6
" 25	10.03	+7.0	" 2	10.67	+5.2	" 21	15.17	+4.1
" 26	10.13	+5.2	" 2	18.84	+6.5	" 22	15.12	+1.7
" 26	10.15	+4.8	" 3	10.71	+4.0	" 23	13.95	+3.2
" 27	10.17	+6.1	" 3	10.79	+5.5	" 23	13.95	+3.5
" 27	10.20	+4.8	" 3	10.96	+3.2	" 24	13.97	+1.6
" 27	10.68	+7.1	" 3	11.14	+2.9	" 25	13.98	+1.8
" 28	10.84	+6.2	" 3	11.33	+4.4	" 26	14.68	—0.5
" 28	10.59	+4.9	" 3	11.38	+3.7	" 27	15.73	+1.2
" 28	11.47	+3.2	" 4	11.00	+3.6	" 27	15.68	+2.1
" 28	11.70	+5.5	" 4	11.33	+4.4	" 28	15.75	+1.1
" 28	11.82	+5.6	" 4	11.55	+2.9	" 28	15.46	—1.0
" 29	9.96	+3.7	" 4	11.68	+5.4	July 1	14.73	+4.1
" 29	10.47	+2.7	" 5	11.22	+3.9	" 2	14.62	+5.4
" 29	12.14	+3.6	" 8	12.12	+5.8	" 3	14.80	+3.5
" 29	10.80	+1.2	" 9	11.75	+4.9	" 4	15.15	+1.9
" 31	10.76	+5.0	" 10	11.78	+4.2	" 5	15.10	+3.5
" 31	10.71	+5.3	" 11	11.78	+3.6	" 6	15.10	+1.0
" 31	10.88	+6.3	" 13	13.13	+3.6	" 7	15.14	+1.9
Jan. 1	10.56	+6.2	" 14	13.49	+4.0	" 8	15.14	+2.0

Arranging the results in groups of 10 each, we have—

From	May 25 to May 27,	R_2—R_3=+5.32μ
"	May 28 to May 31,	+4.16
"	May 31 to June 2,	+4.81
"	June 3 to June 4,	+4.00
"	June 5 to June 16,	+4.05
"	June 16 to June 27,	+2.23
"	June 28 to July 8,	+2.34

If these observations can be trusted, we have here a decided diminution in the value of the relation R_2—R_3, between May and July. It may be added, also, that the evidence of this diminution is still more marked in the comparisons of R_2 with the glass bar G, de-

scribed on page 295 of the Memoir above referred to. In this case the observations extend from March to July, 1885.

Adopting the mean of the values given above, we have—

$$R_3+3.8\mu=R_2=A_0+1.6\mu$$

Or $$R_3+2.2\mu=A_0.$$

The relation previously obtained, is—

$$R_3+2.4\mu=A_0.$$

Until further comparisons have been obtained, the relation,

$$R_3+2.3\mu=A_0.$$

will be adopted.

Since R_2 has defining lines only for the meter and for the half meters, a new set of graduations was laid off upon the edge of this bar. The lines are ruled directly upon the bronze metal, and the sub-divisions are of the same form as upon R_3, with the exception of the last decimeter, which is subdivided to centimeters. This set of graduations is designated $\overset{e}{R}_2$.

Comparison of Meter $\overset{e}{R}_2$ with $\overset{a}{R}{}^2_2$.

With old Comparator.

Date.	$\overset{a}{R}{}^2_2-\overset{e}{R}_2$.
1884, July 19	$+5.9\mu$
" " 19	$+5.3$
" " 19	$+5.1$
1885, Mar. 16	$+4.4$
" " 16	$+4.6$
" " 16	$+4.8$
" " 17	$+5.3$
" " 17	$+5.5$
" " 18	$+4.6$
" " 22	$+3.2$
" " 24	$+4.1$
" " 24	$+2.5$
" " 25	$+3.0$
Mean,	4.49μ

With new Comparator.

Date.	$\overset{a}{R}{}^2_2-\overset{e}{R}_2$.
1885, Apl. 1	$+3.8\mu$
" " 1	$+4.7$
" " 2	$+3.4$
" " 2	$+4.8$
" " 3	$+3.7$
" " 3	$+3.8$
Mean,	4.03μ

Adopt. $\overset{e}{R}_2+4.3\mu=\overset{a}{R}{}^2_2$

Whence:

$$\overset{e}{R}_2 + 2.7\mu = A_0$$

The relation, $\overset{e}{R}_2 - 0.5\mu = Y$, is also given, omitting details.

Sub-divisions of $\overset{e}{R}_2$ and R_3 into Two Equal Parts.

$\frac{1}{2}$ Meters of $\overset{e}{R}_2$.		$\frac{1}{2}$ Meters of R_3.	
Date.	I–II.	Date.	I–II.
1884, June 15	$+1.9\mu$	1884, Oct. 17	$+1.8\mu$
" June 27	$+2.4$	" " 28	-1.9
" July 15	$+2.2$	" " 30	$+0.1$
" July 18	$+1.5$	" " 30	$+2.5$
" Oct. 14	$+2.2$	1885, Jan. 15	$+2.3$
" Oct. 15	$+1.5$	" " 16	-1.8
" Oct. 17	$+1.7$	" " 16	-1.0
" Oct. 17	$+2.6$	" " 19	-1.4
		" " 20	-2.0
		" " 22	-2.2
		" " 22	-1.8
		" " 22	$+1.8$

Whence:

$$\mathrm{I} - 1.0\mu = \tfrac{1}{2}\,\overset{e}{R}_2. \qquad \mathrm{I} + 0.15\mu = \tfrac{1}{2}\,R_3.$$

$$= \tfrac{1}{2}\,A_0 - 1.3\mu \qquad = \tfrac{1}{2}\,A_0 - 1.15\mu$$

And $\mathrm{I} + 0.3\mu = \tfrac{1}{2}\,A_0$ And $\mathrm{I} + 1.3\mu = \tfrac{1}{2}\,A_0$.

Hence, that half of the meter which is sub-divided to decimeters is, in the case of $\overset{e}{R}_2$, 0.3μ *too short*, and in the case of R_3 is 1.3μ *too short*, compared with one-half of the Metre des Archives.

Relative corrections of the decimeter sub-divisions. Separate results of observations made between Feb. 17 and Feb. 23, 1885:

DECIMETERS OF R_3.

1	2	3	4	5
—3.5μ	—2.9μ	+6.4μ	—2.4μ	+2.4μ
—5.0	—2.1	+6.8	—1.8	+2.1
—4.5	—2.6	+7.5	—2.9	+2.4
—5.1	—2 4	+6.0	—2.9	+3.4
—4.5	—2.7	+7.1	—2.5	+2.3
—5.9	—1.5	+7.4	—2.3	+3.1
—5.1	—1.4	+6.1	—0.2	+0.7
—5.3	—1.4	+5.6	—0.9	+2.0
—4.0	—3.2	+6.5	—3.0	+3 6
—5.2	—1.6	+6.9	—2.5	+2.4
—5.4	—2.2	+5.5	—1.2	+3.2
—6.4	—3.0	+6.6	—0.7	+3 5
—5.8	—3.3	+7.6	—1.4	+3.2
—5.3	—2.6	+6.6	—1.4	+2.8
—5.2	—2.6	+7.2	—1.3	+1.8
—6 6	—3.1	+7.3	—0.6	+3.0
—6.3	—3.4	+6.6	—0.6	+3.7
—4.4	—1.8	+6 9	—2.4	+1.7

DECIMETERS OF $\overset{e}{R}_2$.

—0.6μ	+3 2μ	+1.7μ	—0.6μ	—3.6μ
+1.5	+2.8	+0.8	—1.2	—3.8
+1.1	+2.4	+0.1	—2.1	—1.4
+1.2	+1.6	+0.8	—0 3	—3.4
+1.7	+2.3	+ 2.4	—1.4	—4 9
+1 3	+2 8	+1.6	—1.8	—3.8
+1.0	+2.9	+1.9	—2.2	—3 6
—0.1	+2.1	+2.7	—0.6	—4.3
+0.9	+1.7	+2.3	—0.3	—4.7

Taking the means, we have—

Corrections to dms. of R_3. Corrections to dms. of $\overset{e}{R}_2$.

		Σ		Σ
*1	—5.17μ	—5.17μ	+0.89μ	+0.89μ
2	—2.43	—7.60	+2.42	+3.31
3	+6.70	—0.90	+1.59	+4.90
4	+1.72	—2.62	—1.17	+3.73
5	+2.63	+0.00	+3.73	+0.00

The relative corrections for the separate centimeters are given without details, as follows:

Centimeters of R_3.

		Σ
1	—5.6μ	—5.6μ
2	+3.8	—1.8
3	—1.7	—3.5
4	+5.2	+1.7
5	—1.7	+0.0

Centimeters of $\overset{e}{R}_2$.

		Σ
1	—3.4μ	—3.4μ
2	+1.9	—1.5
3	+0.6	—0.9
4	—1.1	—2.0
5	—0.4	—2.4
6	+0.1	—2.3
7	—0.6	—2.9
8	+0.0	—2.9
9	+0.4	—2.5
10	+2.5	+0.0

5 Centimeter Spaces of R_3.

		Σ
1	+5.6μ	+5.6μ
2	—5.6	0.0

Since the first 15 centimeters of $\overset{abc}{S_0}$ will be compared with the first 15 centimeters of R_3, it is necessary to obtain the total amount of the correction for this space. We have—

The second decimeter is relatively too long,	—2.43μ
The second 5 centimeter space of the first decimeter is too long, ...	—5.60μ
One-half of the first decimeter is too long	—2.58μ
The 15 centimeter space is too short, on account of the relation $\mathrm{I}+1.3\mu=\frac{1}{2}\,A_0$	$+0.39\mu=\frac{3}{10}\times 1.3\mu$

* The decimeter at the end of the bar. A plus sign indicates that the measured space is shorter than the mean—a minus sign that it is longer than the mean.

Hence :

The first 15 centimeters of R_3—$10.2\mu = \frac{15}{100}$ A_0.

In the same way it will be found that—

The first decimeter of $\overset{\bullet}{R}_2$	$+1.0\mu = \frac{1}{10}$ A_0.
The first two decimeters of $\overset{\bullet}{R}_2$	$+3.4\mu = \frac{1}{5}$ A_0.
The first decimeter of R_3	$-4.9\mu = \frac{1}{10}$ A_0.
The first two decimeters of R_3	$-7.1\mu = \frac{1}{5}$ A_0.

We must now determine the coefficient of expansion of the speculum metal with all possible precision.

Seven separate series of observations were undertaken for this purpose. They are described as follows:

Series (a) consists of the comparison of a half meter and a half yard $\overset{a}{S}_0$ with the screw W of my dividing engine, which has the coefficient 10.60μ. The bar was supported upon the platen at its neutral points, and was adjusted for level and for line of motion by means of a microscope attached to a carriage which has an independent movement parallel with the screw. The comparisons were made by running the screw carriage to one end, adjusting the micrometer of the microscope for coincidence with the initial line of the standard, and then by carrying the bar forward by means of the screw until the coincidence of the terminal line with the micrometer line of the stationary microscope was made. The difference between the readings for the two positions at one temperature compared with the difference at another temperature, together with the difference of the temperatures, are the data from which the relative coefficient was obtained.

Series (b) consists of a set of comparisons of a half yard and a half meter S made for Professor C. K. Wead of the University of Michigan with standards R_2 and R_3.

Series (c). In this series, half yard and half meter $\overset{a}{S}_0$ were compared with R_3, before June, 1884.

Series (d). Half yard and half meter $\overset{b}{S}_0$ compared with R_2 before before June, 1884.

Series (e). Half yard and half meter $\overset{b}{S}_0$, compared with R_2 between July and November, 1884.

Series (f). Half yard and half meter $\overset{b}{S}_0$ compared with R_3 between July and November, 1884.

Series (g). In this series the comparison of R_3 was made with half meters $\overset{a}{S}_0$, $\overset{b}{S}_0$, $\overset{c}{S}_0$, and also with the other sets of graduations $\overset{d}{S}_0$, $\overset{e}{S}_0$, $\overset{f}{S}_0$, in which the lines are quite heavy. The observations extend from December 12, 1884, to January 23, 1885. In this series a Fahrenheit thermometer was employed, but the readings are reduced to the Yale standard.

The comparisons are all given in the form of equations, each equation representing the mean of all the observations made near the temperature indicated by the coefficient of the unknown quantity (b). The separate values of the quantity (a), which represents the difference in length between the standards compared at 62.00 Fahr., are also given, as well as the residuals (v) from the mean value.

SERIES (a).

Equations of Condition.

½ Meter $\overset{e}{S}_0$ with W.

No. Obs.	W—S_1. (16°.67 F.)	Residuals. a	v
3	$-25.5\mu = a + 4.02b$	-10.7μ	$+2.7\mu$
5	$-27.6 = a + 3.45$	-14.9	-1.5
4	$-21.0 = a + 2.24$	-12.8	$+0.6$
5	$-20.9 = a + 1.19$	-16.5	-3.1
4	$-9.4 = a + 0.14$	-8.9	$+4.5$
4	$-10.3 = a - 0.93$	-13.7	-0.3
5	$-7.7 = a - 1.84$	-14.5	-1.1
3	$+0.1 = a - 4.03$	-14.7	-1.3
4	$+5.6 = a - 4.83$	-12.1	$+1.3$
5	$+7.5 = a - 6.05$	-14.8	-1.4

Normal Equations.

$$-109.2\mu = 10a - 6.64b$$
$$-320.0\mu = -6.64a + 114.94b.$$
$$b = -3.55\mu.$$
$$b = -3.81\mu \text{ from } \tfrac{1}{2} \text{ yard.}$$

Coeff. of W=10.60μ.

Coeff. between W and S_0=3.68μ.

Coeff. of S_0=17.96μ.

EQUATIONS OF CONDITION.

½ Yard S_0 with W.

No. Obs.	$W-S_0$	(16.67° F.)	Residuals. a	v
4	—25.3μ	= a+4.02b	—11.7μ	+0.6μ
4	—26.1	= a+3.45	—14.4	—2.1
3	—22.6	= a+2.13	—15.4	—3.1
4	—20.1	= a+1.22	—16.0	—3.7
5	— 8.1	= a+0.03	— 8.0	+4.3
4	— 5.9	= a—0.73	— 8.4	+3.9
3	— 3.0	= a—1.71	— 8.8	+3.5
4	— 0.7	= a—4.08	—14.5	—2.2
4	+ 2.3	= a—4.74	—13.7	—1.4
5	+ 8.4	= a—6.05	—12.0	+0.3

Normal Equations.

—101.1μ=—10a—6.46b.

—314.1μ=—6.46a+113.26b.

b=3.48μ.

b=3.81μ for ½ meter.

SERIES (b).

½ Meter S with ½ R_2.

No. Obs.	½ R_2—S.	(16.67° F.)	Residuals. a	v
6	+14.0μ	= a+ 6.92b	+11.0μ	—0.3μ
6	+11.9	= a+ 1.40	+11.3	+0.0
6	+10.3	= a— 0.41	+10.5	—0.8
6	+11.3	= a— 3.84	+12.9	+1.9
6	+ 4.9	= a—13.34	+10.6	—0.7

Normal Equations.

+52.40μ=5a—9.27b.

+0.56μ=—9.27a+242.73b.

b=0.43μ.

Coeff. between R_2 and S=0.86μ for 1 meter.

Coeff. of R_2=17.17μ.

Coeff. of S=18.03μ.

½ Yard S with ½ R_3.

No. Obs.	½ R_3—S.	(16.67 F.)	Residuals. a	v
6	+33.0μ	= a+ 6.63b	+10.0μ	—1.0μ
6	+15.5	= a+ 1.38	+10.7	—0.3
6	+ 7.1	= a+ 0.16	+10.1	—0.9
6	+ 0.6	= a— 4.03	+14.6	+3.6
6	—43.5	= a—15.34	+ 9.7	—1.3

Normal Equations.

$$+12.70\mu = 5a - 12.22b.$$

$$+898.94\mu = -12.22a + 298.16b.$$

$$b = 3.47\mu.$$

$$b = 3.79\mu \text{ for } \tfrac{1}{2} \text{ meter.}$$

Coeff. between R_3 and S=7.58μ.

Coeff. of R_3=10.27μ.

Coeff. of S=17.85μ.

SERIES (c).

½ Meter $\overset{a}{S}_0$ with ½ R_3.

No. Obs.	½ R_3—$\overset{a}{S}_0$.	(16.67 F.)	Residuals. a	v
4	+40.6μ	= a+ 8.87b	+ 6.9μ	—1.0μ
4	+21.7	= a+ 3.10	+ 9.9	+2.0
5	+14.2	= a+ 2.14	+ 6.1	—1.8
4	+14.2	= a+ 1.14	+ 9.9	+2.0
3	+ 8.0	= a+ 0.13	+ 7.5	—0.4
2	+10.7	= a+ 0.05	+10.5	+2.6
3	+ 4.0	= a— 1.11	+ 8.2	+0.3
4	—37.0	= a—12.33	+ 9.9	+2.0
5	—54.7	= a—15.09	+ 2.6	—5.3
5	—48.1	= a—14.24	+ 9.8	+1.9
6	—56.2	= a—16.58	+ 6.8	—1.1
5	—59.3	= a—17.48	+ 7.1	—0.8

Normal Equations.

$$-141.9\mu = 12a - 62.40b.$$
$$+4406.03\mu = -62.40a + 1287.87b.$$
$$b = +3.80\mu$$

Coeff. between R_3 and $\dot{S}_0 = 7.60\mu$.

Coeff. of $R_3 = 10.27\mu$.

Coeff. of $\dot{S}_0 = 17.87\mu$.

$\frac{1}{2}$ Yard $\dot{S}_0$ with $\frac{1}{2}$ R_3.

No. Obs.	$\frac{1}{2}R_3 - \dot{S}_0$. (16.67 F.)	Residuals. a	Residuals. v
4	$+38.2\mu = a + 8.85b$	$+7.1\mu$	-3.9μ
4	$+38.8 = a + 8.79$	$+7.9$	-3.1
5	$+30.1 = a + 5.55$	$+10.6$	-0.3
4	$+24.8 = a + 3.10$	$+13.9$	$+3.0$
3	$+17.5 = a + 2.14$	$+12.0$	$+1.0$
3	$+20.3 = a + 2.14$	$+12.8$	$+1.8$
4	$+13.2 = a + 1.45$	$+8.1$	-2.9
4	$+16.8 = a + 1.14$	$+12.8$	$+1.8$
3	$+5.2 = a + 0.12$	$+4.8$	-6.2
3	$+13.3 = a + 0.05$	$+13.1$	$+2.1$
3	$+7.5 = a + 0.03$	$+7.4$	-3.6
4	$+8.7 = a - 0.43$	$+10.2$	-0.8
4	$+6.0 = a - 1.11$	$+9.9$	-1.1
5	$+2.1 = a - 2.91$	$+12.3$	$+1.3$
6	$+2.9 = a - 3.51$	$+15.3$	$+4.3$
4	$+2.9 = a - 3.51$	$+15.3$	$+4.3$
2	$+0.0 = a - 4.29$	$+15.1$	$+4.1$
3	$+0.4 = a - 4.33$	$+15.6$	$+4.6$
4	$-3.1 = a - 4.53$	$+12.8$	$+1.8$
5	$-34.6 = a - 12.33$	$+8.8$	-2.2
6	$-42.4 = a - 15.09$	$+10.7$	-0.3
5	$-41.1 = a - 15.24$	$+12.5$	$+1.5$
4	$-48.2 = a - 16.58$	$+10.2$	-0.8
3	$-57.4 = a - 17.48$	$+4.1$	-7.2

Normal Equations.

$$+2.19\mu = 24a - 67.98b.$$
$$+4524.3\mu = -67.98a + 1493.20b.$$
$$b = 3.52\mu.$$
$$\overset{a}{b} = 3.85\mu \text{ for } \tfrac{1}{2} \text{ meter.}$$

Coeff. between R_3 and $\overset{a}{S_0} = 7.70\mu$.

Coeff. of $R_3 = 10.27\mu$.

Coeff. of $S_0 = 17.97\mu$.

SERIES (d).

$\frac{1}{2}$ Meter $\overset{b}{S_0}$ with $\frac{1}{2}$ R_2.

No. Obs.	$\frac{1}{2}R_2 - \overset{b}{S_0}$. (16.67 F.)	Residuals. a	v
3	$+12.2\mu = a + 8.79b$	$+9.0\mu$	$+1.1\mu$
4	$+8.4 = a + 3.10$	$+7.3$	-0.6
5	$+7.2 = a + 2.10$	$+6.4$	-1.5
4	$+10.0 = a + 1.45$	$+9.5$	$+1.6$
3	$+10.2 = a + 1.14$	$+9.8$	$+1.9$
4	$+9.4 = a + 0.13$	$+9.4$	$+1.5$
3	$+6.6 = a + 0.07$	$+6.6$	-1.3
4	$+8.0 = a - 0.75$	$+8.3$	$+0.4$
3	$+9.2 = a - 1.11$	$+9.6$	$+1.7$
3	$+3.1 = a - 12.33$	$+7.5$	-0.4
4	$+1.2 = a - 15.09$	$+6.6$	-1.3
5	$+0.7 = a - 15.24$	$+6.2$	-1.7
4	$-1.1 = a - 16.58$	$+4.9$	-3.0
4	$+2.7 = a - 17.48$	$+9.0$	$+1.1$

Normal Equations.

$$+87.8\mu = 14a - 61.80b.$$
$$+64.04\mu = 61.80a + 1288.94b.$$
$$b = +0.32\mu.$$

Coeff. of $R_2 = 17.17\mu$.

Coeff. between R_2 and $\overset{b}{S_0} = 0.64\mu$.

Coeff. of $\overset{b}{S_0} = 17.81\mu$.

$\frac{1}{2}$ Yard $\overset{b}{S}_0$ with $\frac{1}{2}$ R_2.

No. Obs.	$R_2 - \overset{b}{S}_0$. (16.67 F.)	Residuals. a	v
3	$+13.4\mu = a + 8\ 87b$	$+10.5\mu$	-0.7μ
4	$+12.8 = a + 8.79$	$+9.9$	-1.3
5	$+15.7 = a + 3.10$	$+14.7$	$+3.5$
5	$+12.6 = a + 2.14$	$+11.9$	$+0.7$
5	$+12.8 = a + 2.14$	$+12.1$	$+0.9$
4	$+13.2 = a + 1.45$	$+12.7$	$+1.5$
5	$+13.4 = a + 1.14$	$+13.0$	$+1.8$
4	$+7.5 = a + 0.13$	$+7.5$	-3.7
5	$+8.8 = a + 0.12$	$+8.5$	-2.7
4	$+9.7 = a + 0.15$	$+9.7$	-1.5
5	$+9.1 = a - 0.43$	$+9.2$	-2.0
5	$+11.7 = a - 0.75$	$+11.9$	$+0.7$
5	$+12.3 = a - 1.11$	$+12.7$	$+1.5$
4	$+11.8 = a - 2.91$	$+12.8$	$+1.6$
4	$+10.9 = a - 3.51$	$+12.1$	$+0.9$
4	$+12.6 = a - 3.51$	$+13.8$	$+2.6$
5	$+11.5 = a - 4.23$	$+12.9$	$+1.7$
4	$+11.6 = a - 4.33$	$+13.0$	$+1.8$
4	$+9.3 = a - 4.53$	$+10.8$	-0.4
4	$+2.2 = a - 12.33$	$+6.3$	-4.9
5	$+5.0 = a - 15.79$	$+10.0$	-1.2
4	$+6.3 = a - 15.24$	$+11.3$	$+0.1$
3	$+6.4 = a - 16.58$	$+11.9$	$+0.7$
4	$+4.6 = a - 17.48$	$+10.4$	-0.8

Normal Equations.

$$+24.49\mu = 24a - 74.00b.$$

$$-297.02\mu = -74.00a + 1462.85b.\mu$$

$b = 0.37\mu$ for $\frac{1}{2}$ yard.

$b = 0.40\mu$ for $\frac{1}{2}$ meter.

Coeff. of $R_2 = 17.17\mu$.

Coeff. between R_2 and $\overset{b}{S}_0 = 0.80\mu$.

Coeff. of $\overset{b}{S}_0 = 17.97$.

SERIES (e).

$\frac{1}{2}$ Meter and $\frac{1}{2}$ Yard $\overset{b}{S}_0$ with $\frac{1}{2}$ R_2.

$\frac{1}{2}$ Meter.

No. Obs.	$R_2—\overset{b}{S}_0$.	(16·67 F.)	Residuals. a	v
5	+18.6div.=	a+10.95b	5.8μ	—1.6μ
8	+15.8 =	a— 1.65	9.0	+1.6
9	+16.2 =	a— 3.44	9.9	+2.5
7	+ 3.0 =	a— 9.57	5.3	—2.1
4	+ 1.6 =	a—16.15	6.9	—0.5

Normal Equations.

+55.40 div.=5a—19.86b.

+65.41 div.=—19.86a+486.85b.

b=—0.70 div.

b=—0.35μ.

Coeff. R_2=17.17μ.

Coeff. between R_2 and $\overset{b}{S}_0$=0.70μ.

Coeff. $\overset{b}{S}_0$=17.87μ.

$\frac{1}{2}$ Yard.

No. Obs.	$R_2—\overset{b}{S}_0$.	(16.67 F.)	Residuals. a	v
5	+17.0 =	a+10.95b	+5.0μ	—1.8μ
8	+13.1 =	a— 1.65	+8.2	+1.4
9	+14.8 =	a— 3.44	+9.1	+2.2
7	+ 6.8 =	a— 9.57	+7.1	+0.3
4	— 2.2 =	a—16.15	+4.8	—2.0

Normal Equations.

+49.5 div.=5a—19.86b.

+84.1 div.=—19.86a+486.85b.

b=—0.69 div.

b=0.35μ for $\frac{1}{2}$ yard.

b=0.28μ for $\frac{1}{2}$ meter.

Coeff. R_2=17.17μ.

Coeff. between R_2 and $\overset{b}{S}_0$= .76μ.

Coeff. S_0=17.93μ.

SERIES (f).

$\frac{1}{2}$ Meter and $\frac{1}{2}$ Yard $\overset{b}{S}_0$ with $\frac{1}{2}$ R_3.

$\frac{1}{2}$ Meter.

No. Obs.	$\frac{1}{2}$ R_3—$\overset{b}{S}_0$.	(16.67 F.)	Residuals. a	v
6	+ 97.7div.	=a+10.95b	+7.5μ	+0.0μ
6	+ 38.2	=a+ 3.49	+6.2	—1.3
6	— 9.5	=a— 3.48	+8.9	+1.4
6	— 72.2	=a—11.09	+6.3	—1.3
6	—116.8	=a—17.35	+8.8	+1.3

Normal Equations.

—62.6 div.=5a—17.48b.

+4063.4 div.=—17.48a+568.20b.

b=7.58 div.

b=3.81μ.

Coeff. of R_3=10.27μ.

Coeff. between R_3 and $\overset{b}{S}_0$=7.62μ.

Coeff. of $\overset{b}{S}_0$=17.89μ.

$\frac{1}{2}$ Yard.

No. Obs.	$\frac{1}{2}$ R_3—$\overset{b}{S}_0$.	(16.67 F.)	Residuals. a	v
6	+ 89.2div.	=a+10.95b	+5.7μ	—1.0μ
6	+ 38.6	=a+ 3.49	+7.2	+0.5
6	— 10.6	=a— 3.48	+7.5	+0.8
6	— 65.6	=a—11.09	+7.4	+0.7
6	—113.1	=a—17.35	+5.8	—0.9

Normal Equations.

—61.5 div.=5a—17.48b.

+383.81 div.=—17.48b+568.2b.

b=7.15 div.=3 60μ.

b=3.94μ for $\frac{1}{2}$ meter.

Coeff. of R_3=10.27μ.

Coeff. between R_3 and $\overset{b}{S}_0$=7.88μ.

Coeff. $\overset{b}{S}_0$=18.15μ.

SERIES (g).

$\frac{1}{2}$ Meter $\overset{a\,b\,c\,d\,e\,f}{S_0}$ with $\frac{1}{2}$ R_3 [in divisions of micrometer.]

No. Obs.	$\frac{1}{2}R_3-\overset{a}{S_0}$	$\frac{1}{2}R_3-\overset{b}{S_0}$	$\frac{1}{2}R_3-\overset{c}{S_0}$	$\frac{1}{2}R_3-\overset{d}{S_0}$	$\frac{1}{2}R_3-\overset{e}{S_0}$	$\frac{1}{2}R_3-\overset{f}{S_0}$	
4	+183.3	+186.6	+205.7	+215.7	+188.3	+190.2	= a+47.59b
4	+166.4	+173.0	+192.4	+205.5	+167.9	+176.7	= a+43.21b
4	+133.7	+138.7	+158.2	+169.8	+137.3	+138.3	= a+34.37b
4	+ 91.4	+ 96.1	+116.8	+127.2	+ 90.1	+ 88.8	= a+24.20b
4	+ 24.4	+ 28.7	+ 48.4	+ 58.7	+ 24.5	+ 33.0	= a+ 7.87b
4	— 95.2	— 98.0	— 79.2	— 70.2	— 99.1	— 94.1	= a—19.23b

Equations of Condition.

From $R_3-\overset{a}{S_0}$.

+84.00=a+23.00b.
—179.28=a+45.87b.
a=—11.9 div.
b=+4.17 div.

From $R_3-\overset{d}{S_0}$.

+117.78=a+23.00b.
+216.44=a+45.87b.
a=+18.6.
b=+4.31.

From $R_3-\overset{b}{S_0}$.

87.52=a+23.00b.
185.19=a+45.87b.
a=—10.3 div.
b=+4.27 div.

From R_3 and $\overset{c}{S_0}$.

+84.83=a+23.00b.
+182.70=a+45.87b.
a=—13.6.
b=+4.28.

From $R_3-\overset{c}{S_0}$.

+107.05=a+23.00b.
204.85=a+45.87b.
a=+9.00.
b=+4.28.

From $R_3-\overset{b}{S_0}$.

+88.82=a+23.00b.
+185.92=a+45.87b.
a=—8.9.
b=+4.25.

Collecting the results of this series we have:

	For 1 Meter.	Equivalent for 1° C.	Coeff. S_0
Coeff. between $\frac{1}{2}$ R_3 and $\overset{a}{S}_0=2.10\mu=$	$4.20\mu=$	7.56μ	17.83μ
" " " $\overset{b}{S}_0=2.15$ =	4.30 =	7.74	18.00
" " " $\overset{c}{S}_0=2.16$ =	4.32 =	7 78	18.04
" " " $\overset{d}{S}_0=2.17$ =	4.33 =	7.81	18.08
" " " $\overset{e}{S}_0=2.16$ =	4.32 =	7.78	18.04
" " " $\overset{f}{S}_0=2.14$ =	4.28 =	7.70	17.97
		Mean	17.99

Combining the results of each series we have:

Coefficient from Series (a)=17.96μ
" " " (b)=17.94μ
" " " (c)=17.92μ
" " " (d)=17.89μ
" " " (e)=17.90μ
" " " (f)=18.02μ
" " " (g)=17.99μ

We have, therefore, finally, for the coefficient of the speculum metal made by Professor Rowland—

17.946μ.

The next step in the operation of standardizing the speculum scales $\overset{s}{S}_0$, $\overset{s}{S}_1$ and $\overset{s}{S}_2$, was their comparison with the correspond-

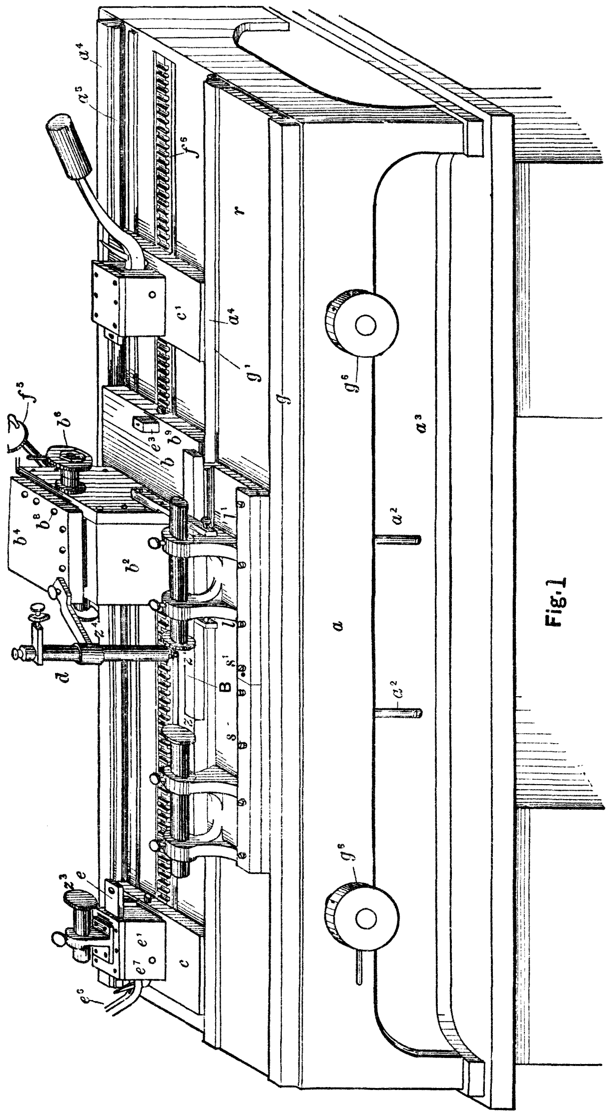

Fig. 1

ing units of R_2 and R_3. In order to secure as great a degree of independence as possible in these comparisons, the conditions under

which they were made were varied by the use of comparators of different forms of construction, which were mounted in different locations, and also by the interchange of the thermometers employed. The old comparator, of which a description is given in my paper, "On Two Forms of Comparators for Measures of Length," is mounted in the new comparing-room beneath the rotunda of the Observatory. The comparator, constructed by the writer in the latter part of 1884, is mounted in the comparing-room situated in the west cellar of the Observatory.

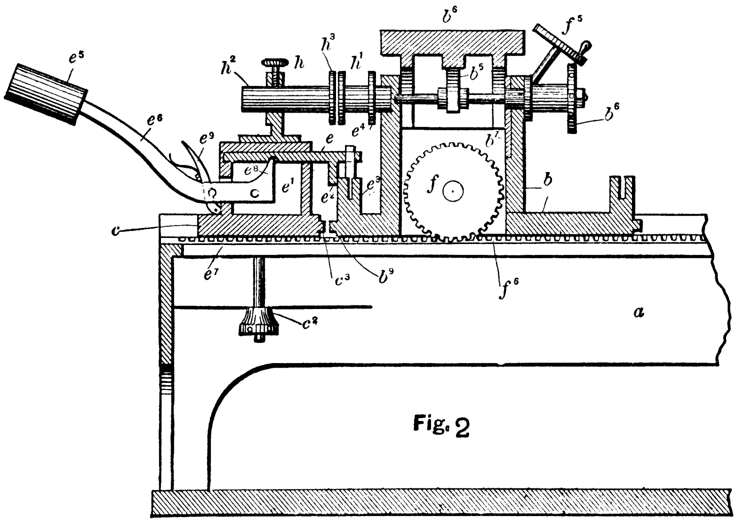

Fig. 2

The main features of this instrument will be seen from the following outline references. In Fig. 1, the microscope slide b^2, which is closely fitted to the projecting side bearings a^4 and a^5 and to similar elevated bearings beneath, is carried the entire length of the bed by the rack f^6 and the bevel gear pinions f^2 f^3 and the pinion f, Fig. 3. The microscope plate b^4 has a slow motion adjustment in elevation by means of an eccentric b^5, Fig. 2.

The stops c c^1 can be set at any desired position upon the bed. They can be firmly secured without the slightest disturbance of the stops, by means of large circular clamps beneath the bed-plate at e_2, Fig. 63.

The plate r extends the entire length of the bed, and is closely

fitted between the walls g and g^1, Fig. 3. It rests upon two eccentrics opposite g^5 and g^6, Fig. 1, and shown in the end view at g^1 g^2, Fig. 64. The adjustment in elevation is made by means of levers inserted in the wheels g^5 g^6.

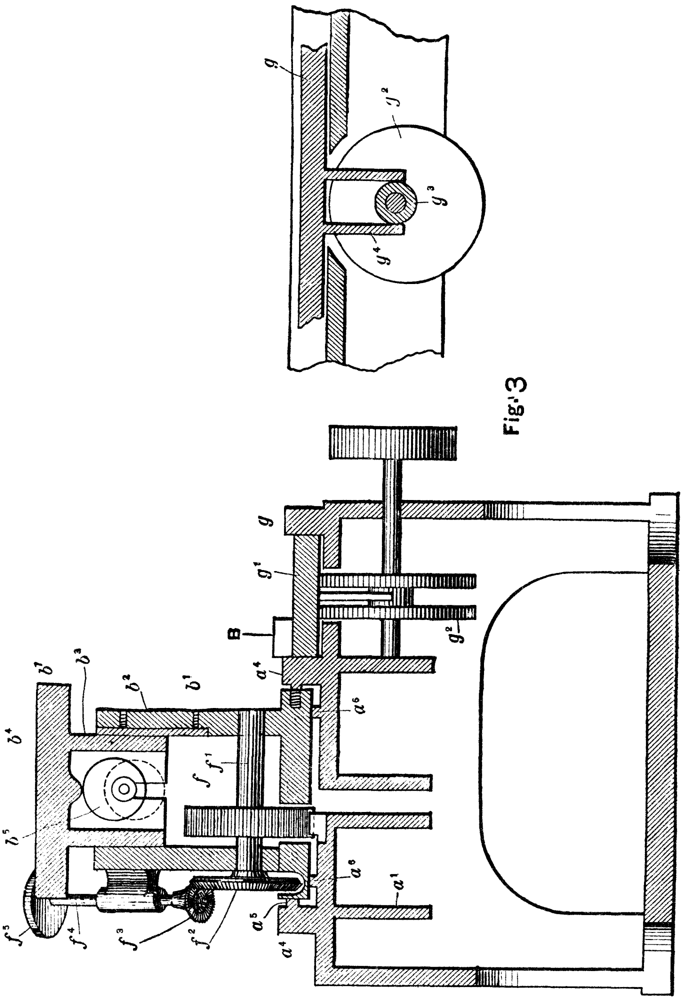

Fig. 3

The gravity lock of the microscope plate against the stops is shown in Fig. 2. The weighted lever e^6 can be thrown out of connection by means of the spring catch e^9, when it is desired to

make the contact with the stops by means of the rack and pinion movement.

The graduated bar *B* rests upon the bed of the machine and against a vertical ledge which extends the entire length.

The universal caliper s s^1 rests upon the plate r, and can be placed in any desired position. The two parts s s^1 move independently; s^1 being carried by two arms attached to the microscope slide b^2.

For convenience, the old comparator will be designated "Comparator A," and the new instrument will be called "Comparator B." The comparing-room in the west cellar of the Observatory will be called "Comparing-room C," and the other, "Comparing-room D."

The thermometers employed are, Farenheit thermometer Baudin 8614, and Centigrade thermometer Y O. The readings given are all reduced to the "Yale Standard." The comparisons are given in terms of the micrometer of the microscope, in which:

$$1 \text{ div.} = .503\mu.$$

An inch objective, supplied with Tolles' illuminator, was used throughout the series.

DATA FROM OBSERVATIONS.

SERIES (1).—Comparison of 15 Centimeters of $\overset{a}{S}_0$ with $\frac{15}{100}$ R_3.

Comparator B.			Comparing-room C.		
		At 62.0° F.			At 62.0° F.
Dates of Observation.	B. 8614.	$[\frac{15}{100} R_3 - \overset{a}{S}_0]$	Dates of Observation.	B. 8614	$[\frac{15}{100} R_3 - \overset{a}{S}_0]$
1885, Jan. 13	17.40°	+16.5 div.	Jan. 25	77.64°	+20.0 div.
" 13	18.10	+22.9	" 26	81.82	+18.8
" 13	17.94	+15.9	" 26	81.84	+18.1
" 13	17.93	+16.6	" 27	31.84	+22.4
" 13	18.11	+19.0	" 27	31.68	+21.0
" 13	28.36	+23.3	" 27	32.14	+18.7
" 13	28.61	+20.9	" 27	33.06	+20.6
" 24	73.10	+22.0	Feb. 1	47.66	+20.5
" 25	74.98	+12.4	" 1	47.80	+20.1
" 25	75.00	+14.5	" 2	43.42	+21.3
" 25	77.60	+17.1	" 3	47.74	+16.6

SERIES (2).—Comparison of the first decimeter of $\overset{a}{S}_0$, $\overset{b}{S}_0$ and $\overset{c}{S}_0$ with the mean of decimeters 1, 2, 3, 4, 5 of R_3.

Comparator B. Comparing-room C.

Dates of Observation.	Y O	$\frac{1}{10} R_3 - \overset{a}{S}_0$.	$\frac{1}{10} R_3 - \overset{b}{S}_0$.	$\frac{1}{10} R_3 - \overset{c}{S}_0$.
Feb. 17	7.35°	+0.4 div.	+0.4 div.	+3.8 div.
" 17	7.25	—3.1	+0.4	+3.1
" 18	5.81	+1.7	+4.0	+7.7
" 18	7.45	+1.1	+0.8	+4.0
" 19	5.56	+1.6	+0.6	+5.5
" 19	6.56	—2.2	+0.0	+2.9
" 19	6.95	—5.7	—2.9	+2.3
" 19	4.06	+0.1	—0.9	+2.2

At 16.67° C.

Dates of Observation.	Y O	[$\frac{1}{10} R_3 - \overset{a}{S}_0$.	$\frac{1}{10} R_3 - \overset{b}{S}_0$.	$\frac{1}{10} R_3 - \overset{c}{S}_0$.]
Feb. 22	21.98°	—2.7 div.	—1.7 div.	+2.5 div.
" 22	20.80	—1.8	—2.1	—0.1
" 22	22.52	—2.7	—1.0	+2.0
" 22	22.32	—2.3	—0.3	+3.4
" 22	22.42	—0.5	+1.3	+5.5
" 23	21.00	—4.7	—2.6	+0.1
" 23	21.98	—2.3	—1.5	+2.7
" 24	20.78	+0.3	+1.0	+4.9

SERIES (3).—Comparison of decimeter $R_2 \frac{12\ 845}{5}$ with $\overset{a}{S}_0$, $\overset{b}{S}_0$, $\overset{c}{S}_0$.

Comparator B. Comparing-room C.

At 16.67° C.

Dates of Observation.	Y O	[$\frac{1}{10} R_2 - \overset{a}{S}_0$.	$\frac{1}{10} R_2 - \overset{b}{S}_0$.	$\frac{1}{10} R_2 - \overset{c}{S}_0$.]
Feb. 24	22.72°	—1.4 div.	+1.2 div.	+4.7 div.
" 24	23.70	—1.9	+0.7	+3.6
" 25	26.51	—3.1	—1.3	+2.4
" 25	26.21	—0.6	+0.3	+2.9
" 26	25.68	—2.4	—1.6	+0.2
" 27	14.39	—0.6	+2.7	+2.6
" 27	14.19	—0.1	+4.0	+7.0
" 27	14.48	+0.1	+3.7	+4.2

SERIES (4).—Comparison of two decimeters of $\overset{a}{S}_1$ with $\frac{1}{5}$ R_3.

Comparator A.		At 62.0° F.	Comparing-room D.		At 62.0° F.
Dates of Observation.	B. 8614.	[$\frac{1}{5}$ R_3—$\overset{a}{S}_1$.]	Dates of Observation.	B. 8614	[$\frac{1}{5}$ R_3—$\overset{a}{S}_1$.]
Feb. 24	28.20°	+15.6 div.	Feb. 26	29.10°	+18.9 div.
" 24	28.40	+20.0	" 26	29.30	+20.7
" 25	30.36	+16.9	" 27	30.06	+19.7
" 25	30.20	+16.8	" 27	30.22	+18.7
" 25	29.48	+15.1	" 27	30.64	+16.3
" 25	30.26	+18.2	" 27	30.44	+17.2
" 26	30.30	+16.5	" 27	31.06	+15.1

SERIES (5).—Comparison of two decimeters of S_1 with $\frac{1}{5}$ R_2.

Comparator B.		At 16.67° C.	Comparing-room C.		At 16.67° C.
Dates of Observation.	Y O	[$\frac{1}{2}$ R_3—$\overset{a}{S}_1$.]	Dates of Observation.	Y O	[$\frac{1}{5}$ R_3—$\overset{a}{S}_1$.]
Mar. 2	24.35°	+16.5 div.	Mar. 3	23.95°	+16.3 div.
" 2	24.84	+18.0	" 4	14.39	+12.2
" 3	24.25	+20.3	" 4	13.58	+10.7
" 3	24.25	+13.6	" 6	14.68	+13.4
" 3	24.24	+11.7	" 8	11.56	+20.7
" 3	23.54	+19.6	" 8	11.71	+19.9
" 3	24.14	+11.1	" 8	11.41	+16.1
" 3	25.13	+23.2	" 8	11.41	+15.9
" 3	24.34	+13.8	" 9	11.86	+18.9
" 3	24.84	+18.5	" 9	11.81	+16.3
" 3	24.44	+15.1	" 9	11.91	+22.0
" 3	24.76	+19.8	" 9	11.91	+21.0
" 3	23.95	+15.9	" 10	8.63	+14.8
" 3	24.74	+20.9	" 10	8.53	+14.2

SERIES (6).—Comparison of two decimeters of S_1 with $\frac{1}{5}$ R_2.

Comparator B.		At 62.0° F.	Comparing-room C.		At 62.0° F.
Dates of Observation.	B. 8614.	[$\frac{1}{5}$ R_2—$\overset{a}{S}_1$.]	Dates of Observation.	B. 8614.	[$\frac{1}{5}$ R_2—$\overset{a}{S}_1$.]
Mar. 10	31.76°	—8.8 div.	Mar. 11	29.64°	—9.3 div.
" 10	31.44	—8.3	" 12	30.44	—8.0
" 11	29.87	—6.9	" 12	31.26	—4.6
" 11	30.80	—6.5	" 13	26.14	—6.5

SERIES (7) —Comparison of 2 decimeters of S_1 with $\frac{1}{5}$ R_3.

Comparator B.		At 62.0° F.	Comparing-room C.		At 62.0° F.
Dates of Observation.	Y O	$[\frac{1}{5} R_3 - \overset{a}{S}_1.]$	Dates of Observation.	B. 8614.	$[\frac{1}{5} R_2 - \overset{a}{S}_1.]$
Mar. 13	48.13°	+12.6 div.	Mar. 15	22.57°	+15.5 div.
" 13	50.81	+16.2	" 15	22.96	+17.8
" 13	48.67	+11.3	" 15	23.06	+17.6
" 15	23.15	+20.6	" 15	22.86	+11.3
" 15	22.32	+18.8	" 15	22.96	+19.5
" 15	22.18	+15.7	" 15	23.01	+19.4

SERIES (8).—Comparison of two decimeters of R_3 with $\overset{a}{S}_2$.

Comparator B.		At 16.67° C.	Comparing-room C.		At 62.0° F.
Dates of Observation.	Y O	$[\frac{1}{5} R_3 - \overset{a}{S}_2]$	Dates of Observation.	B. 8614.	
Mar. 10	8.83°	+16.7 div.	Mar. 13	48.13°	+10.8 div.
" 10	9.43	+12.0	" 13	50.81	+15.9
" 10	9.33	+12.5	" 13	48.67	+19.9
" 11	9.93	+11.7	" 15	23.15	+19.0
" 11	10.13	+ 9.1	" 15	22.32	+15.3
" 11	10.23	+12.8	" 15	22.18	+13.0
" 11	10.42	+11.4	" 15	22.57	+13.1
" 12	7.64	+ 8.8	" 15	22.96	+16.5
" 12	7.94	+10.6	" 15	23.06	+16.0
" 12	10.82	+17.9	" 15	22.86	+18.4
" 12	12.01	+18.1	" 15	22.96	+17.6
" 13	8.48	+ 9.4	" 15	23.01	+19.5
" 13	8.68	+10.6			

SERIES (9).—Comparison of the decimeters of $\overset{a}{S}_1$ with $R_8 \frac{12345}{5}$.

Comparator B.

At 16.67° C.

Dates of Observation.	Y O	$[\frac{1}{10} R_8 - \overset{a}{S}_1.]$	
	°	I	II
Feb. 17	7.35	+1.0 div.	+1.8 div.
" 17	7.25	—7.5	—7.4
" 18	5.81	+4.2	+5.0
" 18	7.45	+5 5	+6.9
" 19	5.56	+1.3	+2.0
" 19	7.35	+4.0	+4.8
" 19	6.56	—2.8	—3.8
" 19	6.95	—1.4	+.06

Comparing-room C.

At 16 67° C.

Dates of Observation.	Y O	$[\frac{1}{10} R_8 - \overset{a}{S}_1.]$	
	°	I	II
Feb. 19	4.06	—2.3 div.	—0.5 div.
" 20	4.98	—3.1	+0.2
" 22	20.80	—0.2	+0.1
" 22	22.52	—2.9	—0.8
" 22	23.32	—3.6	—4.2
" 22	22.42	—3.6	—1.7
" 23	21.00	—6.7	—3.6
" 23	21.98	—2.9	+0.4
" 24	20.78	—1.2	+1.1

SERIES (10).—Comparison of the first and the second decimeters of $\overset{a}{S}_2$ with $R_0 \frac{12345}{5}$.

Comparator B.

At 16.67° C.

Dates of Observation.	Y O	$[\frac{1}{10} R_3 - \overset{a}{S}_2.]$	
	°	I	II
Feb. 17	7.35	—0.6 div.	—0.5 div.
" 17	7.25	—4.4	—3.4
" 18	5.81	+3.7	+4.8
" 18	7.45	+4.2	+4.3
" 19	5.56	—1.8	—1.0
" 19	7.35	+4.5	+5.3
" 19	6.56	—3.8	—4.9
" 19	6.95	—3.9	—3.2

Comparing-room C.

At 16.67° C.

Dates of Observation.	Y O	$[\frac{1}{10} R_3 - \overset{a}{S}_2.]$	
	°	I	II
Feb. 19	4.06	+1.1 div.	+1.2 div.
" 22	21.98	—1.4	—1.4
" 22	20.80	—2.2	—0.5
" 22	22.52	—1.8	—2.9
" 22	23.32	—3.6	—3.0
" 22	22.42	—3.4	—2.1
" 23	21.00	—5.0	—4.4
" 23	21.98	—1.2	—0.8
" 24	20.78	—1.6	—1.0

SERIES (11).—Comparison of the first and second decimeters of $\overset{a}{S}_2$ with $R_2 \frac{12345}{5}$.

Comparator B.				Comparing-room C.			
Dates of Observation.	Y O	$[\frac{1}{10} R_2 - \overset{a}{S}_2.]$		Dates of Observation.	Y O	$[\frac{1}{10} R_2 - \overset{a}{S}_2.]$	
	°	I	II		°	I	II
Feb. 24	22.72	+0.6 div.	+0.7 div.	Feb. 26	25.68	+2.5 div.	+2.6 div.
" 24	23.70	—2.3	—1.8	" 27	14.39	+2.5	+3.2
" 25	26.51	+0.4	+0.4	" 27	14.19	+2.1	+5.4
" 25	26.24	+0.1	—0.7	" 27	14.48	+2.1	+3.1

SERIES (12).—Comparison of the first and second decimeters of $\overset{a}{S}_2$ with $R_2 \frac{12345}{5}$.

Comparator A. At 62.0° F.				Comparing-room D. At 62.0° F.			
Dates of Observation.	B. 8614.	$[\frac{1}{10} R_3 - \overset{a}{S}_2.]$		Dates of Observation.	B. 8614.	$[\frac{1}{10} R_3 - \overset{a}{S}_2.]$	
	°	I	II		°	I	II
Mar. 1	32.98	—3.5 div.	—3.8 div.	Mar. 4	32.22	—1.0 div.	+0.0 div.
" 2	32.48	—2.7	—2.8	" 5	32.96	—0.7	+4.1
" 2	32.06	—2.1	—2.1	" 5	32.78	—1.7	+1.4
" 3	32.46	+0.5	+0.9	" 6	32.00	—3.9	—2.5
" 3	32.20	+0.2	+2.0	" 8	31.66	+0.5	+4.1
" 3	32.20	—1.1	+1.3	" 9	27.62	+2.8	+5.7
" 4	32.54	—2.7	+1.8				

SERIES (13).—Comparison of the decimeters of S^a_1 and S^a_2.

Dates of Observation.	EQUATION BETWEEN I and II of S^a_1.	EQUATION BETWEEN I and II of S^a_2.
Mar. 16	I—0.7μ= $\frac{1}{2} S^a_1$	I—0.2μ= $\frac{1}{2} S^a_2$.
" 16	I—0.3 = $\frac{1}{2} S^a_1$	I—0.2 = $\frac{1}{2} S^a_2$.
" 16	I—0.6 = $\frac{1}{2} S^a_1$	I—0.2 = $\frac{1}{2} S^a_2$.
" 18	I—0.4 = $\frac{1}{2} S^a_1$	I—0.4 = $\frac{1}{2} S^a_2$.
" 18	I—0.6 = $\frac{1}{2} S^a_1$	I—0.4 = $\frac{1}{2} S^a_2$.
" 19	I—0.1 = $\frac{1}{2} S^a_1$	I—0.3 = $\frac{1}{2} S^a_2$.
" 19	I—0.1 = $\frac{1}{2} S^a_1$	I—0.3 = $\frac{1}{2} S^a_2$.
" 19	I—0.2 = $\frac{1}{2} S^a_1$	I—0.2 = $\frac{1}{2} S^a_2$.
" 23	I—0.3 = $\frac{1}{2} S^a_1$	I—0.2 = $\frac{1}{2} S^a_2$.
" 23	I—0.1 = $\frac{1}{2} S^a_1$	I—0.3 = $\frac{1}{2} S^a_2$.
" 24	I+0.0 = $\frac{1}{2} S^a_1$	I—0.4 = $\frac{1}{2} S^a_2$.
" 24	I—0.1 = $\frac{1}{2} S^a_1$	I—0.7 = $\frac{1}{2} S^a_2$.
" 25	I—0.1 = $\frac{1}{2} S^a_1$	I—0.4 = $\frac{1}{2} S^a_2$.
" 25	I+0.0 = $\frac{1}{2} S^a_1$	I—0.2 = $\frac{1}{2} S^a_2$.
" 25	I—0.3 = $\frac{1}{2} S^a_1$	I—0.4 = $\frac{1}{2} S^a_2$.
" 26	I+0.2 = $\frac{1}{2} S^a_1$	I—0.1 = $\frac{1}{2} S^a_2$.
" 26	I+0.2 = $\frac{1}{2} S^a_1$	I—0.2 = $\frac{1}{2} S^a_2$.

SERIES (14).—Comparison of the five centimeter spaces of $\overset{a}{S}_1$ and $\overset{a}{S}_2$.

Dates of Observation.	Corrections to five centimeter spaces of $\overset{a}{S}_1$.			
	1	2	3	4
Feb. 1	—0.8μ	—0.1μ	+0.6μ	—0 2μ.
" 16	—0.4	—0.1	+0.6	—0.2
Mar. 19	—0.1	+0.2	+0.9	—1.0
" 22	—0.5	+0.1	+1.2	—0.8
" 22	—0.3	—0.3	+0.5	+0.0
" 23	—0.5	—0.1	+0.8	+0.1
" 24	—0.1	+0.1	+0.8	—0.6
" 25	—0.8	+0.1	+0.9	—0.1
" 26	+0.2	+0.2	+0.6	—0.8
" 26	—0.4	—0.5	+0.9	—0.4

Dates of Observation.	Corrections to five centimeter spaces of $\overset{a}{S}_2$.			
	1	2	3	4
Feb. 1	—0.2μ	+0.1μ	+0.8μ	—0.8μ.
" 16	—0.5	—0 2	+0.6	+0.2
Mar. 19	+0.0	+0.0	+0.4	—0.4
" 22	—0.3	+0.9	+0.6	—1.2
" 22	—0.2	+0.2	+0.8	—0.6
" 23	+0.6	+0.0	+0.4	—0.8
" 24	—0.1	—0.2	+0.4	—0.1
" 25	—0.2	+0.1	+0.5	—0.5
" 26	—0.5	—0.4	+1.0	—0.3
" 26	—0.7	+0.0	+0.6	+0 0

Collecting results we have:

From Series (13).

$$\text{First decimeter of } \overset{a}{S}_1 - 0.19\mu = \tfrac{1}{2}\,\overset{a}{S}_1.$$

$$\text{First decimeter of } \overset{a}{S}_2 - 0.30\mu = \tfrac{1}{2}\,\overset{a}{S}_2.$$

From Series (14).

Correction of five centimeter spaces of $\overset{a}{S}_1$.

		Σ
1	-0.37μ	-0.37μ.
2	-0.01μ	-0.38μ.
3	$+0.78\mu$	$+0.40\mu$.
4	-0.40μ	$+0.00\mu$.

Corrections to five centimeter spaces of $\overset{a}{S}_2$.

		Σ
1	-0.21μ	-0.21μ.
2	$+0.05\mu$	-0.16μ.
3	$+0.61\mu$	$+0.45\mu$.
4	-0.45μ	$+0.00\mu$.

It will be seen that the corrections to the decimeter spaces derived from the summation of the corrections to the five centimeter spaces are 0.38μ and 0.16μ, respectively. Taking the mean between these values and the values found by direct comparison we have:

$$\text{First decimeter of } \overset{a}{S}_1 - 0.28\mu = \tfrac{1}{2}\,\overset{a}{S}_1.$$

$$\text{First decimeter of } \overset{a}{S}_2 - 0.23\mu = \tfrac{1}{2}\,\overset{a}{S}_2.$$

SERIES (15.) Direct comparison of $\overset{a}{S}_0$, $\overset{b}{S}_0$ and $\overset{c}{S}_0$.

Dates of Observation.	$\overset{a}{S}_0-\overset{b}{S}_0$.	$\overset{a}{S}_0-\overset{c}{S}_0$.	$\overset{b}{S}_0-\overset{c}{S}_0$.
Mar. 30	$+0.3\mu$	$+1.7\mu$	$+1.4\mu$
" 30	$+0.7$	$+3.2$	$+2.5$
Ap'l 1	$+0.4$	$+2.3$	$+1.9$
" 1	-0.1	$+2.6$	$+2.7$
Means	$+0.32\mu$	$+2.45\mu$	$+2.12\mu$

SERIES (16).

Direct comparison of five centimeter spaces of $\overset{a}{S}_0$, $\overset{b}{S}_0$ and $\overset{c}{S}_0$.

Dates of Observation.	$\overset{a}{S}_0$. 1	2	3
Mar. 29	$+1.1\mu$	-0.6μ	-0.5μ
" 30	$+0.8$	-0.5	-0.3
" 31	$+1.1$	-0.7	-0.4
Means	$+1.00\mu$	-0.60μ	-0.40μ

Dates of Observation.	$\overset{b}{S}_0$. 1	2	3	$\overset{c}{S}_0$. 1	2	3
Mar. 29	-0.0μ	$+0.5\mu$	-0.5μ	-0.5μ	$+1.2\mu$	-0.7μ.
" 30	$+0.4$	$+0.5$	-0.9	-0.9	$+1.8$	-0.9
" 31	$+0.1$	$+0.8$	-0.9	-0.7	$+1.5$	-0.8
Means	$+0.17\mu$	$+0.57\mu$	-0.74μ	-0.70μ	$+1.50\mu$	-0.80μ

SERIES (17).—Direct comparison of the two decimeters of $\overset{a}{S}_1$ and of $\overset{a}{S}_2$.

Dates of Observation.	$\overset{a}{S}_1-\overset{a}{S}_2$.	Dates of Observation.	$\overset{a}{S}_1-\overset{a}{S}_2$.
April 1	-0.8μ.	April 2	-1.1μ.
" 1	-1.1	" 2	-0.6
" 1	-0.8	" 3	-0.4
" 1	-0.4	" 3	-1.4
" 1	-1.2	" 3	-1.4
" 1	-1.0		

Hence: $\overset{a}{S}_1-\overset{a}{S}_2=-0.93\mu$.

Independent determinations of the relations between the decimeters of $\overset{a\,b\,c}{S}_0$, $\overset{a}{S}_1$, $\overset{a}{S}_2$ and $\frac{1}{10}$ A_0.

From a combination of the results obtained from Series (c) to Series (g), pages 170 to 176, we have:

$$\overset{a}{S}_0-6.3\mu=\tfrac{1}{2}\,R_3\,(I)=\tfrac{1}{2}\,A_0-1.3\mu \text{ and } \overset{a}{S}_0-5.0\mu=\tfrac{1}{2}\,A_0.$$

$$\overset{b}{S}_0-5.3\mu=\tfrac{1}{2}\,R_3\,(I)=\tfrac{1}{2}\,A_0-1.3\mu \qquad \overset{b}{S}_0-4.0\mu=\tfrac{1}{2}\,A_0.$$

$$\overset{c}{S}_0+4.8\mu=\tfrac{1}{2}\,R_3\,(I)=\tfrac{1}{2}\,A_0-1.3\mu \qquad \overset{c}{S}_0+6.1\mu=\tfrac{1}{2}\,A_0.$$

Hence, disregarding the relative errors of the sub-divisions we have:

First decimeter of $\overset{a}{S}_0$—$1.0\mu = \frac{1}{10}\ A_0$.

First decimeter of $\overset{b}{S}_0$—$0.8\mu = \frac{1}{10}\ A_0$.

First decimeter of $\overset{c}{S}_0 + 1.2\mu = \frac{1}{10}\ A_0$.

Referring to page 190 and combining the relative corrections of the first decimeter of $\overset{a}{S}_0$, $\overset{b}{S}_0$ and $\overset{c}{S}_0$, with the relations given above, we have for the total corrections to the first decimeters of S_0:

$\frac{1}{5}\ \overset{a}{S}_0 + 0.3\mu - 1.0\mu = \frac{1}{10}\ A_0$, or $\frac{1}{5}\ \overset{a}{S}_0 - 0.7\mu = \frac{1}{10}\ A_0$.

$\frac{1}{5}\ \overset{b}{S}_0 + 1.2\mu - 0.8\mu = \frac{1}{10}\ A_0$, $\quad \frac{1}{5}\ \overset{b}{S}_0 + 0.4\mu = \frac{1}{10}\ A_0$.

$\frac{1}{5}\ \overset{c}{S}_0 + 0.4\mu + 1.2\mu = \frac{1}{10}\ A_0$, $\quad \frac{1}{5}\ \overset{c}{S}_0 + 1.6\mu = \frac{1}{10}\ A_0$.

From Series (1), page 181, we have:

15 centimeters of $\overset{a}{S}_0 - 0.9\mu = \frac{3}{20}\ A_0$.

From page 189,

$\overset{a}{S}_0 - \overset{b}{S}_0 = +0.3\mu. \qquad \overset{a}{S}_0 - \overset{c}{S}_0 = +2.5\mu.$

Hence: $\frac{3}{10}\ \overset{b}{S}_0 - 0.6\mu = \frac{3}{20}\ A_0$. $\quad \frac{3}{10}\ \overset{c}{S}_0 + 1.6\mu = \frac{3}{20}\ A_0$.

From Series (16), and from the relation between $\overset{a\,b\,c}{S_0}$ and $\frac{1}{2}\ A_0$ given on page 190, we have:

Spaces.	Corrections to $\overset{a}{S}_0$.	Σ
1	$+1.0\mu - 0.5\mu = +0.5\mu$	$+0.5\mu$.
2	$-0.6 \ -0.5 \ = -1.1$	-0.6
3	$-0.4 \ -0.5 \ = -0.9$	-1.5

Spaces.	Corrections to $\overset{b}{S}_0$.	Σ	Corrections to $\overset{c}{S}_0$.	Σ
1	$+0.2\mu - 0.4\mu = -0.2\mu$	-0.2μ.	$-0.7\mu + 0.6\mu = -0.1\mu$	-0.1μ.
2	$+0.6 \ -0.4 \ = +0.2$	$+0.0$	$+1.5 \ +0.6 \ = +2.1$	$+2.0$
3	$-0.7 \ -0.4 \ = -1.1$	-1.1	$-0.8 \ +0.6 \ = -0.2$	$+1.8$

We have, therefore:

For the first 5 centimeters.	For the first decimeter.	For the first 15 centimeters.
$\frac{1}{10}\overset{a}{S}_0+0.5\mu=\frac{1}{20}A_0.$	$\frac{1}{10}\overset{b}{S}_0-0.2\mu=\frac{1}{20}A_0.$	$\frac{1}{10}\overset{c}{S}_0-0.1\mu=\frac{1}{20}A_0.$
$\frac{1}{5}\overset{a}{S}_0-0.6\mu=\frac{1}{10}A_0.$	$\frac{1}{5}\overset{b}{S}_0+0.0\mu=\frac{1}{10}A_0.$	$\frac{1}{5}\overset{c}{S}_0+2.0\mu=\frac{1}{10}A_0.$
$\frac{3}{10}\overset{a}{S}_0-1.5\mu=\frac{3}{20}A_0.$	$\frac{3}{10}\overset{b}{S}_0-1.1\mu=\frac{3}{20}A_0.$	$\frac{3}{10}\overset{c}{S}_0+1.8\mu=\frac{3}{20}A_0.$

From Series (2) and Series (3) we have:

From comparison with R_3.

$$\frac{1}{5}\overset{a}{S}_0-0.7\mu=\frac{1}{10}R_3=\frac{1}{10}A_0-0.2\mu.$$
$$\frac{1}{5}\overset{b}{S}_0-0.2\mu=\frac{1}{10}R_3=\frac{1}{10}A_0-0.2\mu.$$
$$\frac{1}{5}\overset{c}{S}_0+1.7\mu=\frac{1}{10}R_3=\frac{1}{10}A_0-0.2\mu.$$

From comparison with R_2.

$$\frac{1}{5}\overset{a}{S}_0-0.6\mu=\frac{1}{10}R_2=\frac{1}{10}A_0-0.2\mu.$$
$$\frac{1}{5}\overset{b}{S}_0+0.6\mu=\frac{1}{10}R_2=\frac{1}{10}A_0-0.2\mu.$$
$$\frac{1}{5}\overset{c}{S}_0+1.8\mu=\frac{1}{10}R_2=\frac{1}{10}A_0-0.2\mu.$$

Whence:

$\frac{1}{5}\overset{a}{S}_0-0.5\mu=\frac{1}{10}A_0.$	$\frac{1}{5}\overset{a}{S}_0-0.4\mu=\frac{1}{10}A_0.$
$\frac{1}{5}\overset{b}{S}_0+0.0\mu=\frac{1}{10}A_0.$	$\frac{1}{5}\overset{a}{S}_0+0.8\mu=\frac{1}{10}A_0.$
$\frac{1}{5}\overset{c}{S}_0+1.9\mu=\frac{1}{10}A_0.$	$\frac{1}{5}\overset{b}{S}_0+2.0\mu=\frac{1}{10}A_0.$

Collecting the separate results we have the following corrections for the first decimeters of $\overset{abc}{S}_0$:

Corrections

For $\frac{1}{5}\overset{a}{S}_0$.	For $\frac{1}{5}\overset{b}{S}_0$.	For $\frac{1}{5}\overset{c}{S}_0$.
-0.9μ	-0.6μ	$+1.6\mu$
-0.6μ	$+0.0\mu$	$+2.0\mu$
-0.5μ	$+0.0\mu$	$+1.9\mu$
$-04.\mu$	$+0.8\mu$	$+2.0\mu$

Or, finally:

$$\tfrac{1}{5}\overset{a}{\mathbf{S}}_0 - 0.60\mu = \tfrac{1}{10}\mathbf{A}_0. \quad \tfrac{1}{5}\overset{b}{\mathbf{S}}_0 + 0.05\mu = \tfrac{1}{10}\mathbf{A}_0. \quad \tfrac{1}{5}\overset{c}{\mathbf{S}}_0 + 1.88 = \tfrac{1}{10}\mathbf{A}_0.$$

And for the first five centimeters:

$$\tfrac{1}{10}\overset{a}{\mathbf{S}}_0 + 0.5\mu = \tfrac{1}{20}\mathbf{A}_0. \quad \tfrac{1}{10}\overset{b}{\mathbf{S}}_0 - 0.2\mu = \tfrac{1}{20}\mathbf{A}_0. \quad \tfrac{1}{10}\overset{c}{\mathbf{S}}_0 - 0.1\mu = \tfrac{1}{20}\mathbf{A}_0.$$

For the relation between the two decimeters of $\overset{a}{S}_1$ in terms of $\frac{1}{10}$ A_0 we have:

From Series (4), $\overset{a}{S}_1 + 1.1\mu = \frac{1}{5}A_0$.

From Series (5), $\overset{a}{S}_1 + 1.4\mu = \frac{1}{5}A_0$.

From Series (6), $\overset{a}{S}_1 + 0.2\mu = \frac{1}{5}A_0$.

From Series (7), $\overset{a}{S}_1 + 1.2\mu = \frac{1}{5}A_0$.

Hence $$\tfrac{2}{5}\overset{a}{\mathbf{S}}_1 + 0.98\mu = \tfrac{1}{5}\mathbf{A}_0.$$

Noting the relations between the first and the second decimeters given on page 189 we have:

First decimeter of $\overset{a}{S}_1 + 0.2\mu = \frac{1}{10}A_0$.

From Series (9). First decimeter of $\overset{a}{S}_1 - 0.1\mu = \frac{1}{10}A_0$.

Hence, finally:

First decimeter of $\overset{a}{S}_1 + 0.05\mu = -\frac{1}{10}A_0$.

First 5 centimeter space of $\overset{a}{S}_1 - 0.1\mu = \frac{1}{20}A_0$.

For the relation between the two decimeters of S_2 and $\frac{1}{10}$ A_0 we have:

From Series (8), $\frac{2}{5}\overset{a}{S}_2 + 0.2\mu = \frac{1}{5}A_0$.

Hence:

	First decimetor of			$\overset{a}{S}_2 + 0.0\mu = \frac{1}{10}A_0$.
From Series (10),	"	"	"	$-0.3\mu = \frac{1}{10}A_0$.
From Series (10),	"	"	"	$-0.4\mu = \frac{1}{10}A_0$.
From Series (11),	"	"	"	$+0.5\mu = \frac{1}{10}A_0$.
From Series (11),	"	"	"	$+0\ 6\mu = \frac{1}{10}A_0$.
From Series (12),	"	"	"	$-0.5\mu = \frac{1}{10}A_0$.
From Series (12),	"	"	"	$+0.2\mu = \frac{1}{10}A_0$.

Hence, finally:

$$\textbf{First decimeter of } \overset{a}{S}_2 - 0.01\mu = \tfrac{1}{10}\ A_0.$$

$$\textbf{First 5 centimeter space of } \overset{a}{S}_2 - 0.2\mu = \tfrac{1}{20}\ A_0.$$

It must be carefully noted that all of these relations hold for 6.20° F., or for 16.67° C.

Comparison of the Rowland gratings with $\overset{a}{S}_0$, $\overset{b}{S}_0$, $\overset{c}{S}_0$, $\overset{a}{S}_1$ and $\overset{a}{S}_2$.

The decimeter gratings are designated $\overset{1}{R}_1$, $\overset{2}{R}_1$, $\overset{3}{R}_1$, $\overset{4}{R}_1$. and the five centimeter gratings are designated $\overset{1}{R}_2$, $\overset{2}{R}_2$, $\overset{3}{R}_2$, $\overset{4}{R}_2$.

Dates of Observation.	$\overset{a}{S}_0 - \overset{1}{R}_1$.	$\overset{b}{S}_0 - \overset{1}{R}_1$.	$\overset{c}{S}_0 - \overset{1}{R}_1$.	$\overset{a}{S}_1 - \overset{1}{R}_1$.	$\overset{a}{S}_2 - \overset{1}{R}_1$.
Feb. 1	+127.3div.	+125.4div.	+121.9div.	+128.7div.	+129.0div.
2	128.1	125.3	121.9	129.0	129.5
3	128.8	127.8	125.5	128.8	130.5
4	123.9	128.9	127.2	127.5	127.7
5	127.2	128.6	128.6	129.8	132.5
6	130.6	126.2	123.9	129.8	130.5
6	129.1	126.6	124.2	127.8	131.4
7	129.8	124.9	120.2	128.4	126.2
17	130.5	127.0	124.3	134.9	131.4
18	128.1	123.6	120.1	123.6	125.1

	$\overset{a}{S}_0 - \overset{2}{R}_1$.	$\overset{b}{S}_0 - \overset{2}{R}_1$.	$\overset{c}{S}_0 - \overset{2}{R}_1$.	$\overset{a}{S}_1 - \overset{2}{R}_1$.	$\overset{a}{S}_2 - \overset{2}{R}_1$.
Feb. 1	+130.3div.	+128.4div.	+124.9div.	+131.7div.	+132.0div.
2	130.6	127.8	124.4	131.5	132.0
3	127.9	126.9	124.6	127.9	129.6
4	128.1	126.1	126.4	131.7	131.9
5	129.8	129.2	124.2	132.4	135.1
6	129.3	124.9	122.6	128.5	129.2
6	129.8	127.3	124.9	128.5	132.1
7	129.7	125.0	126.3	128.5	126.3
17	129.8	126.3	123.6	134.2	130.7
18	130.3	125.8	122.3	125.8	127.3

Dates of Observation.	$S^a_0-R^3_1$.	$S^b_0-R^3_1$.	$S^c_0-R^3_1$.	$S^a_1-R^3_1$.	$S^a_2-R^3_1$.
Feb. 1	+129.0div.	+127.1div.	+125.6div.	+130.4div.	+130.7div.
2	128.1	125.3	121.9	129.0	129.5
3	128.4	127.4	125.1	128.4	130.1
4	128.5	126.5	121.8	132.1	132.3
5	129.8	124.2	124.2	132.4	130.1
6	127.5	128.1	126.8	126.7	127.4
6	131.8	129.3	126.9	130.5	134.1
7	131.0	126.1	121.4	129.6	127.4
17	130.3	126.8	124.1	134.7	131.2
18	129.5	125.0	121.5	125.0	126.5

	$S^a_0-R^4_1$.	$S^b_0-R^4_1$.	$S^c_0-R^4_1$.	$S^a_1-R^4_1$.	$S^a_2-R^4_1$.
Feb. 1	+127.7div.	+125.8div.	+122.3div.	+129.1div.	+129.4div.
2	128.7	125.9	122.5	129.6	130.1
3	126.0	125.0	122.7	126.0	127.7
4	129.3	127.3	122.6	127.9	127.1
5	127.3	126.7	126.7	129.9	127.6
6	128.8	124.4	122.1	128.0	128.7
6	129.5	127.0	124.6	128.2	131.8
7	131.4	126.5	121.8	130.0	127.8
17	128.3	124.8	122.1	132.7	129.2
18	129.3	124.8	121.3	124.8	126.3

Dates of Observation.	$S^a_0-R^1_2$.	$S^b_0-R^1_2$.	$S^c_0-R^1_2$.	$S^a_1-R^1_2$	$S^a_2-R^1_2$.
Feb. 6	+66.2 div.	+69.0 div.	+68.8 div.	+68.5 div.	+67.3 div.
7	64.7	66.3	65.9	67.7	67.9
7	62.2	65.4	64.4	63.8	63.7
8	64.3	66.5	65.9	64.7	64.5
9	63.1	64.1	62.8	65.9	64.9
12	62.2	63.4	63.6	63.9	64.3
13	65.3	67.0	64.4	68.3	68.3
15	67.8	70.7	69.3	70.3	72.0
15	64.7	67.0	65.0	64.9	66.5
16	65.6	67.5	68.1	68.5	68.4

	$S_0^a - R_2^2$.	$S_0^b - R_2^2$.	$S_0^c - R_2^2$.	$S_1^a - R_2^2$.	$S_2^a - R_2^2$.
Feb. 6	+64.0 div.	+66.8 div.	+66.6 div.	+66.3 div.	+65.1 div.
7	62.8	64.4	64.0	65.8	66.0
7	62.7	65.9	64.9	64.3	64.2
8	64.4	66.6	66.0	64.8	64.6
9	62.2	63.2	61.9	65.0	64.0
12	61.4	62.6	62.8	63.1	63.5
13	62.0	63.7	61.1	65.0	65.0
15	63.6	66.5	65.1	66.1	67.8
15	64.5	66.8	64.8	64.7	66.3
16	63.6	65.5	66.1	66.5	66.4

Dates of Observation.	$S_0^a - R_2^3$.	$S_0^b - R_2^3$.	$S_0^c - R_2^3$.	$S_1^a - R_2^3$.	$S_2^a - R_2^3$.
Feb. 6	+63.9 div.	+66.7 div.	+66.5 div.	+66.2 div.	+65.0 div.
7	62.9	64.5	64.1	65.9	66.1
7	61.8	65.0	64.0	63.4	63.3
8	65.3	67.5	66.9	65.7	65.5
9	65.6	66.6	65.3	68.4	67.4
12	65.5	66.7	66.9	67.2	67.6
13	63.2	64.9	62.3	66.2	66.2
15	66.0	68.9	67.5	68.5	70.2
15	63.8	66.1	64.1	64.0	65.6
16	65.0	66.9	67.5	67.9	67.8

	$S_0^a - R_2^4$.	$S_0^b - R_2^4$.	$S_0^c - R_2^4$.	$S_1^a - R_2^4$.	$S_2^a - R_2^4$.
Feb. 6	+67.8 div.	+70.6 div.	+70.4 div.	+70.1 div.	+68.9 div.
7	65.0	66.6	66.2	68.0	68.2
7	67.9	71.1	70.1	69.5	69.4
8	65.7	67.9	67.3	66.1	65.9
9	64.9	65.9	64.6	67.7	66.7
12	65.0	66.2	66.4	66.7	67.1
13	63.9	65.6	63.0	66.9	66.9
15	66.8	69.7	68.3	69.3	71.0
15	64.0	66.3	64.3	64.2	65.8
16	63.7	65.6	66.2	66.6	66.5

We have now, as the result of our inquiry:

$\overset{a}{S}_0 - \overset{1}{R}_1 = +128.3$ div. $= +64.5\mu$ whence $\overset{1}{R}_1 + 63.9\mu = \frac{1}{10} A_0$.

$\overset{b}{S}_0 - \overset{1}{R}_1 = +126.4 \quad = +63.6 \quad \overset{1}{R}_1 + 63.6 = \frac{1}{10} A_0$.

$\overset{c}{S}_0 - \overset{1}{R}_1 = +123.8 \quad = +62.3 \quad \overset{1}{R}_1 + 64.2 = \frac{1}{10} A_0$.

$\overset{a}{S}_1 - \overset{1}{R}_1 = +128.8 \quad = +64.8 \quad \overset{1}{R}_1 + 64.7 = \frac{1}{10} A_0$.

$\overset{a}{S}_2 - \overset{1}{R}_1 = +129.4 \quad = +65.1 \quad \overset{1}{R}_1 + 65.1 = \frac{1}{10} A_0$.

Therefore: $\mathbf{\overset{1}{R}_1 + 64.30\mu = \frac{1}{10} A_0}$.

$\overset{a}{S}_0 - \overset{2}{R}_1 = +129.6$ div. $= +65.2\mu$ whence $\overset{2}{R}_1 + 64.6\mu = \frac{1}{10} A_0$.

$\overset{b}{S}_0 - \overset{2}{R}_1 = +126.8 \quad = +63.8 \quad \overset{2}{R}_1 + 63.8 = \frac{1}{10} A_0$.

$\overset{c}{S}_0 - \overset{2}{R}_1 = +124.4 \quad = +62.6 \quad \overset{2}{R}_1 + 64.5 = \frac{1}{10} A_0$.

$\overset{a}{S}_1 - \overset{2}{R}_1 = +130.1 \quad = +65.4 \quad \overset{2}{R}_1 + 65.3 = \frac{1}{10} A_0$.

$\overset{a}{S}_2 - \overset{2}{R}_1 = +130.8 \quad = +65.9 \quad \overset{2}{R}_1 + 65.9 = \frac{1}{10} A_0$.

Therefore: $\mathbf{\overset{2}{R}_1 + 64.82\mu = \frac{1}{10} A_0}$.

$\overset{a}{S}_0 - \overset{3}{R}_1 = +129.4$ div. $= +65.1\mu$ whence $\overset{3}{R}_1 + 64.5\mu = \frac{1}{10} A_0$.

$\overset{b}{S}_0 - \overset{3}{R}_1 = +126.6 \quad = +63.7 \quad \overset{3}{R}_1 + 63.8 = \frac{1}{10} A_0$.

$\overset{c}{S}_0 - \overset{3}{R}_1 = +123.9 \quad = +62.4 \quad \overset{3}{R}_1 + 64.3 = \frac{1}{10} A_0$.

$\overset{a}{S}_1 - \overset{3}{R}_1 = +129.9 \quad = +65.4 \quad \overset{3}{R}_1 + 65.3 = \frac{1}{10} A_0$.

$\overset{a}{S}_2 - \overset{3}{R}_1 = +129.3 \quad = +65.0 \quad \overset{3}{R}_1 + 65.0 = \frac{1}{10} A_0$.

Therefore: $\mathbf{\overset{3}{R}_1 + 64.58\mu = \frac{1}{10} A_0}$.

$\overset{a}{S}_0 - \overset{4}{R}_1 = +128.6$ div. $= +64.7\mu$ whence $\overset{4}{R}_1 + 64.1\mu = \frac{1}{10} A_0$.

$\overset{b}{S}_0 - \overset{4}{R}_1 = +125.8 \quad = +63.3 \quad \overset{4}{R}_1 + 63.4 = \frac{1}{10} A_0$.

$\overset{c}{S}_0 - \overset{4}{R}_1 = +122.9 \quad = +61.8 \quad \overset{4}{R}_1 + 63.7 = \frac{1}{10} A_0$.

$\overset{a}{S}_1 - \overset{4}{R}_1 = +128.6 \quad = +64.7 \quad \overset{4}{R}_1 + 64.7 = \frac{1}{10} A_0$.

$\overset{a}{S}_2 - \overset{4}{R}_1 = +128.6 \quad = +64.7 \quad \overset{4}{R}_1 + 64.7 = \frac{1}{10} A_0$.

Therefore: $\mathbf{\overset{4}{R}_1 + 64\ 12\mu = \frac{1}{10} A_0}$.

$\overset{a}{S}_0 - \overset{1}{R}_2 = +64.6$ div. $= +32.5\mu$ whence $\overset{5}{R}_1 + 33.0\mu = \frac{1}{20}A_0$.
$\overset{b}{S}_0 - \overset{1}{R}_2 = +66.7 \quad = +33.6 \quad \overset{5}{R}_1 + 33.4 = \frac{1}{20}A_0$.
$\overset{c}{S}_0 - \overset{1}{R}_2 = +65.8 \quad = +33.1 \quad \overset{5}{R}_1 + 33.0 = \frac{1}{20}A_0$.
$\overset{a}{S}_1 - \overset{1}{R}_2 = +66.6 \quad = +33.5 \quad \overset{5}{R}_1 + 33.4 = \frac{1}{20}A_0$.
$\overset{a}{S}_2 - \overset{1}{R}_2 = +66.8 \quad = +33.6 \quad \overset{5}{R}_1 + 33.4 = \frac{1}{20}A_0$.

Therefore: $\mathbf{\overset{1}{R}_2 + 33.24\mu = \frac{1}{20} A_0}$.

$\overset{a}{S}_0 - \overset{2}{R}_2 = +63.1$ div. $= +31.7\mu$ whence $\overset{6}{R}_1 + 32.2\mu = \frac{1}{20}A_0$.
$\overset{b}{S}_0 - \overset{2}{R}_2 = +65.2 \quad = +32.8 \quad \overset{6}{R}_1 + 32.6 = \frac{1}{20}A_0$.
$\overset{c}{S}_0 - \overset{2}{R}_2 = +64.3 \quad = +32.3 \quad \overset{6}{R}_1 + 32.1 = \frac{1}{20}A_0$.
$\overset{a}{S}_1 - \overset{2}{R}_2 = +65.2 \quad = +32.8 \quad \overset{6}{R}_1 + 32.7 = \frac{1}{20}A_0$.
$\overset{a}{S}_2 - \overset{2}{R}_2 = +65.3 \quad = +32.8 \quad \overset{8}{R}_1 + 32.6 = \frac{1}{20}A_0$.

Therefore: $\mathbf{\overset{2}{R}_2 + 32.44\mu = \frac{1}{20} A_0}$.

$\overset{a}{S}_0 - \overset{3}{R}_2 = +64.3$ div. $= +32.3\mu$ whence $\overset{7}{R}_1 + 32.8\mu = \frac{1}{20}A_0$.
$\overset{b}{S}_0 - \overset{3}{R}_2 = +66.4 \quad = +33.4 \quad \overset{7}{R}_1 + 33.2 = \frac{1}{20}A_0$.
$\overset{c}{S}_0 - \overset{8}{R}_2 = +65.5 \quad = +32.9 \quad \overset{7}{R}_1 + 32.8 = \frac{1}{20}A_0$.
$\overset{a}{S}_1 - \overset{3}{R}_2 = +66.3 \quad = +33.3 \quad \overset{7}{R}_1 + 33.1 = \frac{1}{20}A_0$.
$\overset{a}{S}_2 - \overset{3}{R}_2 = +66.5 \quad = +33.4 \quad \overset{7}{R}_1 + 33.2 = \frac{1}{20}A_0$.

Therefore: $\mathbf{\overset{3}{R}_2 + 33.02\mu = \frac{1}{20} A_0}$.

$\overset{a}{S}_0 - \overset{4}{R}_2 = +65.5$ div. $= +32.9\mu$ whence $\overset{8}{R}_1 + 33.4\mu = \frac{1}{20}A_0$.
$\overset{b}{S}_0 - \overset{4}{R}_2 = +67.5 \quad = +34.0 \quad \overset{8}{R}_1 + 33.8 = \frac{1}{20}A_0$.
$\overset{c}{S}_0 - \overset{4}{R}_2 = +66.7 \quad = +33.6 \quad \overset{8}{R}_1 + 33.5 = \frac{1}{20}A_0$.
$\overset{a}{S}_1 - \overset{4}{R}_2 = +67.5 \quad = +34.0 \quad \overset{8}{R}_1 + 33.9 = \frac{1}{20}A_0$.
$\overset{a}{S}_2 - \overset{4}{R}_2 = +67.6 \quad = +34.0 \quad \overset{8}{R}_1 + 33.8 = \frac{1}{20}A_0$.

Therefore: $\mathbf{\overset{4}{R}_2 + 33.68\mu = \frac{1}{20} A_0}$.